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Frequency criterion for assessing the stability of digital automatic devices

An analogue of the Mikhailov frequency criterion for assessing the stability of digital automatic systems is considered.

Introduction. Stability is the ability of technical systems to return to its equilibrium position after the termination of the external forces that brought it out of balance. The main methods for determining the stability of linear automatic systems are the root methods, based on solving their linear differential equations, and special methods based on the using of the so-called sustainability criteria.

Most often researchers use in their work frequency criteria of the stability. With their help it is enough simply to make a decision about measures to ensure sustainability of the linear systems, if the result of investigation suggests that it is unstable.

Formulation of the problem. In cases when from the automatic system requires complex information processing or the performance of such operations that cannot be performed with the required accuracy by using linear (analog) devices, they switch to a discrete (digital) automation system.

We note that frequency criteria of linear systems are not suitable for discrete systems and methods for analyzing their stability are not applicable. In each specific case the engineer must choose the method or criterion that is most suitable for analyzing the discrete system.

In our opinion, the analogue of Mikhailov's frequency criterion for studying the stability of discrete systems can be used with the same success as Mikhailov's frequency criterion for linear systems, and the practice of organizing and conducting research dictates the need for its inclusion in simulation packages.

The solution of the problem. An advantage of the analogue of Mikhailov's criterion is the possibility of determining the stability of both closed and open discrete systems.

If in the characteristic polynomial of the discrete system

$$A\left[z\right] = a_0 z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n = 0$$

replaced z by $e^{j\omega T} = \cos\omega T \pm j\sin\omega T$, then by the form of the complex function

$$A\left[e^{j\omega T}\right] = X\left[\omega\right] + jY\left[\omega\right], \tag{1}$$

where $X\left[\omega\right] = f_1\left[a_n + a_i\cos\omega T\right]$, $Y\left[\omega\right] = f_2\left[a_i\sin\omega T\right]$ when the frequency ω is varied, one can judge the stability of the system under study.

Assuming the roots z_i of the characteristic equation to be known, we write the complex function (1) in the form

$$A\left[e^{j\omega T}\right] = a_0\left(e^{j\omega T} - z_1\right)\left(e^{j\omega T} - z_2\right)\ldots\left(e^{j\omega T} - z_n\right). \tag{2}$$

In (2), each factor is a vector of a complex variable $\overline{\left(e^{j\omega T} - z_i\right)} = \overline{e^{j\omega T}} - \overline{z_i}$.

As an example, Figure 1 shows two vectors of a complex variable. Figure 1a shows the vector $\overline{\left(e^{j\omega T}-z_i\right)}$ when the root $|z_i|<1$ is inside the unit circle. In Figure 4b, the position of the vector $\overline{\left(e^{j\omega T}-z_i\right)}$ is determined by a root $|z_i|>1$ located outside the unit circle.

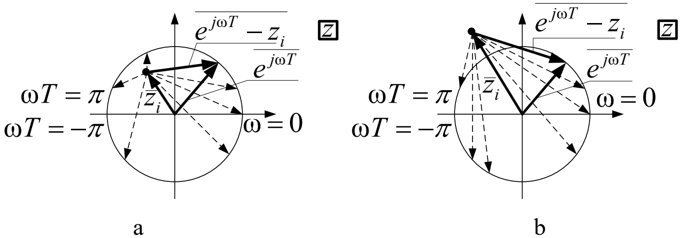

Fig.1 Rotation of the vector $\overline{\left(e^{j\omega T}-z_i\right)}$: a - $|z_i|<1$; b - $|z_i|>1$

Let us investigate the behavior of vectors with a change in frequency in the interval $-\frac{\pi}{T}\le\omega\le+\frac{\pi}{T}$ determined by the frequency of the frequency transfer function of the discrete system.

For $|z_i|<1$ (Figure 1a), the vector $\overline{\left(e^{j\omega T}-z_i\right)}$, sliding along the circumference of the unit radius, makes on the z -plane relative to the end of the vector z_i a complete revolution in the positive direction; turns on an angle $\beta=2\pi$.

For $|z_i|>1$ (Figure 1b), the vector $\overline{\left(e^{j\omega T}-z_i\right)}$, sliding along the circumference of the unit radius, makes on the z -plane relative to the end of the vector z_i a complete revolution in the positive direction; turns on an angle $\beta=0$.

By the rule of multiplication of complex numbers, the total angle of rotation of the vector $A\left[e^{j\omega T}\right]$ is defined as the algebraic sum of the angles $\overline{\left(e^{j\omega T}-z_i\right)}$ of rotation of the vectors that are its factors (2).

Consequently, if all the roots of the characteristic equation are located inside the unit circle $|z_i|<1$, which corresponds to a stable discrete system, the vector $A\left[e^{j\omega T}\right]$ will rotate by an angle $2\pi n$ opposite the clockwise direction.

Restricting ourselves to the study of a stable discrete system in the frequency range $0\le\omega\le\frac{\pi}{T}$, i.e. $0\le\omega T\le\pi$, we obtain the total angle of rotation of the vector $A\left[e^{j\omega T}\right]$ equal to $\frac{\pi}{2}\cdot 2n$.

For the system unstable $|z_i|>1$, the total angle of rotation of the vector $A\left[e^{j\omega T}\right]$ will be zero.

Proceeding from the foregoing, we formulate an analogue of Mikhailov's stability criterion: for stability of a discrete system of n - order one it is necessary and sufficient that when the angular frequency ω changes from 0 to $\frac{\pi}{T}$, the curve described by the end of the vector $A\left[e^{j\omega T}\right]$ passes counterclockwise successively $2n$ quadrants.

Figure 2a shows Mikhailov's hodograph for a discrete stable third-order system.

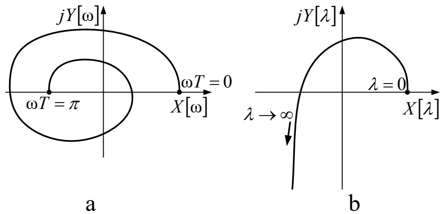

Fig 2 Hodographs Mikhailova for stable discrete third-order system:
a - the angular frequency; b-pseudo-frequency

The study of the complex function $A\left[e^{j\omega T}\right]=X[\omega]+jY[\omega]$ and the construction of the hodograph can be performed using a bilinear transformation.

The algorithm for calculating a variable w using the transformation $z=e^{j\omega T}$ is known

$$w=\frac{z-1}{z+1}=\frac{e^{j\omega T}-1}{e^{j\omega T}+1}=j\mathrm{tg}\frac{\omega T}{2}=j\lambda\frac{T}{2},$$

where $\lambda=\frac{2}{T}\mathrm{tg}\frac{\omega T}{2}$ pseudo-frequency.

We note that the pseudo-frequency has the dimension of the angular frequency. In addition, when ω varies from 0 to $\frac{\pi}{T}$ pseudo-frequency λ changes from 0 to ∞, and when $\omega<\frac{2}{T}$ pseudo-frequency practically coincides with the angular frequency $\lambda\cong\omega$.

Taking into account the bilinear transformation, we have

$$A[j\lambda]=X[\lambda]+jY[\lambda].$$

With the transition to the pseudo-frequency, the Mikhailov hodograph takes the form of the hodographs of a continuous system, and the formulation of the stability criterion for discrete systems is a classical form of Mikhailov's criterion.

Figure 2b shows Mikhailov's travel time graph for a stable discrete third-order system.

Conclusions.

- The frequency criteria of Mikhailov is a serious tool in the study of the stability of discrete automatic devices.
- Every engineer engaged in the research or design of discrete and digital systems must possess these criteria.
- In our opinion, Mikhailov's criterion of stability for discrete systems should be included in visual modeling packages

References

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