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Mathematical modeling in the investigations of interaction of the reinforcement with concrete

The article presents the analytical approach to the analysis the stress-strain state of reinforcement under the conditions of interaction with high-strength concrete. The developed theory will allow engineers-designers to solve problems optimize the design of pavement of airfields and road pavement.

Mathematical model for analyse of stress-deformed state of reinforcing rod in the conditions of pulling out this rod from concrete

The rigid coating of the aerodrome directly perceives the load from the influence of aircraft and operational equipment. Multilayer roads pavement of highways perceives load from vehicles of different types. In both cases, the state of road pavement also significantly depends on additional accompanying mechanical factors and on climatic conditions. At the same time, the full consideration of all indicated factors during the design of rigid road pavement of aerodromes and highways does not allow to predict their durability at the stage of operation [1].

There are various reasons for road pavement damage. The decisive factor for ensuring the strength of the design is the mechanical interaction of fittings with concrete, which depends on the size and shape of the cross-section, profile of the outer surface of the fittings, as well as from the mechanical properties of materials. The comprehensive study of the influence of these indicators is carried out by an experimental study of the task of pulling out the reinforcing rod from concrete [2].

At the stage of designing road clothing designs, the strength and reliability of which increase with reinforced elements, an important task is to evaluate the compatible operation of the fitting with concrete with subsequent optimization of the input parameters. To find the optimal parameters of road pavement constructions it is expedient to conduct mathematical modeling of interaction a reinforcing rod with concrete.

Consider the task of the analysis of stress-deformed state of reinforcing rod in the conditions of pulling out this rod from concrete is considered. The calculated scheme of this task is given in [3]. We consider a simplified calculation scheme a reinforcing rod that interacts with a concrete block. The length of the interaction of the rod and the block is l. Two identical oppositely directed forces F act on the rod and on the block. We will denote N_a the normal force that acts on the reinforcing rod in an anchor area. A normal force in concrete is denoted by N_b. We have:

$$N_b = F - N_a .$$

To determine the analytical dependence of $N_a(x)$, the distribution of normal forces along the length of the rod will consider the constructive element of the rod, the

length dx of which is assumed to be sufficiently small. For the intensity of the axial force in the rod we obtain:

$$\frac{d^2 N_a}{dx^2} = c\left(\frac{N_a}{E_a A_a} - \frac{N_b}{E_b A_b}\right).$$ (1)

Here c – the empirical proportionality factor,

   E_a, E_b – modulus of elasticity of reinforcing rod and concrete,

   A_a, A_b – cross-sectional area of the reinforcing rod and concrete block.

   The differential equation (1) we written in standard form:

$$\frac{d^2 N_a}{dx^2} - k^2 N_a = c \cdot \frac{F}{E_b A_b}.$$ (2)

   When solution the equation (1), we take into account the boundary conditions:

$$N_a(0) = 0, \quad N_a(l) = F .$$ (3)

Analytical dependencies for the distribution of normal stresses on the length of a reinforcing rod

   The total solution of linear inhomogeneous differential equation (3) we can search in the form:

$$N_a(x) = C_1 \, sh(kx) + C_2 \, ch(kx) + \frac{c}{k^2} \cdot \frac{F}{E_b A_b}.$$ (4)

   The integration constants C_1 and C_2 in determine from the limiting conditions (3) of the boundary value problem.

   In (4) the coefficient c is empirical, and the integration constants C_1 and C_2 depends on constructive material of reinforcing rod. For fiberglass fittings, analytical dependence (4) allows approximation in the form:

$$N_a(x) = F \cdot \frac{sh(kx)}{sh(kl)}.$$ (5)

Simulation results

   Having received an analytical dependence for the distribution of normal stresses along the length of the reinforcing rod, we carry out numerical simulation by means of MAPLE Software.

   Numerical experiments show that at small values of the coefficient c graph $N_a(x)$ are almost linear. At $c \geq 10^5$ distribution $N_a(x)$ is nonlinear. In Fig. 1 we have the partition of the normal force along the length of the reinforcing rod. Graphs are built for fiberglass fittings, the use of which for road pavement provides greater

strength and durability of the structure. In Fig. 1, the graph 1 shows an analytical dependence (4), and the graph 2 is constructed for the main part (5) of this dependence.
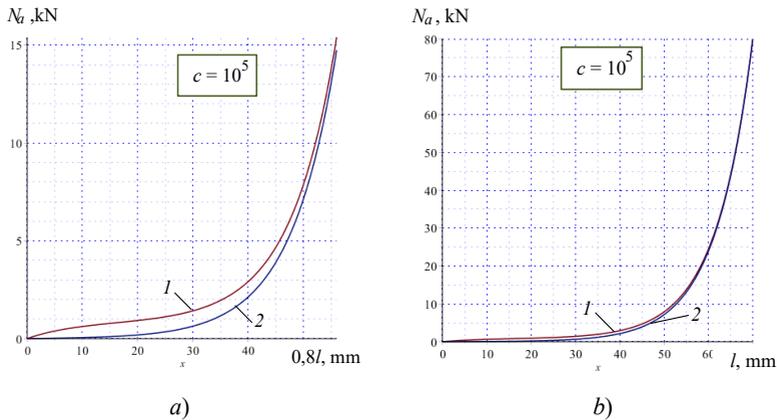

Fig 1. The distribution of normal forces acting in the reinforcing rod

Comparative analysis Fig. 1, a) and Fig. 1, b) show that the differences between (3) and (4) are insignificant and observed only for small values x.

Conclusions

Purposeful experimental and theoretical studies of coupling of fittings with concrete are the basic basis for optimizing the design of modern road pavement in order to increase its reliability and durability. In the present study, analytical dependencies are proposed that make it possible to predict the magnitude of gripping efforts with concrete, as well as to calculate the stresses that arise in reinforcing rods. The found solution of this problem can be used as a basis for comparison with the results of other calculation methods, for example, numerical methods of mechanics of a deformable solid.

References

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