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## Kalman filter for channel estimation in OFDM systems

This paper deals with the estimation of the channel frequency response in communication systems with orthogonal frequency division multiplexing (OFDM) technology. The Kalman filter for the channel frequency response estimation and interpolation in time domain is proposed.

The orthogonal frequency division multiplexing (OFDM) technology is widely used in digital communication systems. This technology allows provide high data rates and effectively use the frequency spectrum [1]. The essence of the technology is to partition the serial data stream into parallel sub-streams for simultaneous transmission at orthogonal frequencies. OFDM signal is a sequence of symbols in time. For the formation of one symbol the inverse fast Fourier transform (IFFT) is applied over the N-dimensional complex array, which is an image of the OFDM symbol in the frequency domain. Elements of complex array are considered as parameters of N-subcarriers, each of which is a point on a modulation constellation (e.g., QPSK, QAM16, QAM64, etc.).

During the pass through the wireless communication channel the signal is influenced by phenomenon of multipath propagation that is negative for narrowband signals [2] and broadband signals. If the communication system is mobile, the signal is exposed to the Doppler effect, which also negatively for it. So, as the subcarriers are modulated by M-positional phase manipulation or quadrature modulation of high orders, the receiver needs to know the exact characteristics of the communication channel for the correct reception of data. To do this, the signal OFDM includes subcarriers are called pilot subcarriers and provide an opportunity to estimate the characteristics of the communication channel.

The Fig. 1 describes the structure of OFDM signal that is widely used in modern standards of digital television. On the basis of it, we can say that the estimation of the channel characteristics – the channel frequency response (CFR) – is divided into two stages: estimation of the values of the frequency response on the pilot subcarriers and interpolation on the data subcarriers.

There are two basic methods for estimation the CFR on the pilot subcarriers: the least square (LS) and the minimum mean square error (MMSE) [1]. The method of linear interpolation, low-pass interpolation and interpolation with using the fast Fourier transform (FFT) are widely used for interpolation CFR on data subcarriers.

In this paper we focus on using the Kalman filter for estimates of the timevarying multipath CFR and interpolate it to the followed data symbols at the pilot subcarriers until the next pilot symbol OFDM symbol is received.

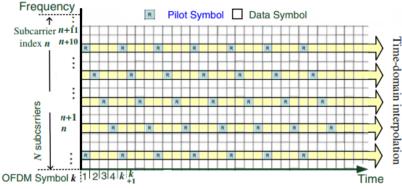


Fig. 1. The structure of OFDM signal

The vector of channel frequency response (CFR) coefficients that changes in time can be modeled by the following dynamic autoregressive (AR) process [3]

$$h_{k+1} = A_k h_k + v_k, \tag{1}$$

where  $A_k$  – the state transition matrix;  $v_k$  – vector of Gaussian complex white noise. The vector of received pilots can be expected as

$$y_k = X_k h_k + w_k, \tag{2}$$

where  $X_k$  – diagonal matrix with transmitted pilot signals of k OFDM symbol as its diagonal elements;  $w_k$  – vector of Gaussian complex white noise.

Considering equations (1) and (2), the CFR at pilot subcarriers can be described as a next model

$$\begin{cases} a_{k+1} = a_k + \varepsilon_k \\ h_{k+1} = A(a_k)h_k + v_k \\ y_k = X_k h_k + w_k \end{cases}$$
(3)

where  $a_k$  – vector of unknown entries of matrix A that describes time correlation of the channel response between k and k+1 OFDM symbols;  $\varepsilon_k$  – vector of Gaussian white noise. In model (3) the vector of state  $h_k$  and vector of parameters  $a_k$  needs to be estimated together. For joint estimation a new augmented state is added, which is defined as

$$z_k = \begin{bmatrix} a_k^T & h_k^T \end{bmatrix}^T.$$
(4)

Thus the model (3) turns into

$$\begin{cases} z_{k+1} = f(z_k) + u_k \\ y_k = [0 \quad X_k] z_k + w_k \end{cases}$$
(5)

where  $u_k = [\varepsilon_k^T \quad v_k^T]^T$  and  $f(z_k)$  – the nonlinear state transition function that defined as

$$f(z_k) = \begin{bmatrix} a_k \\ A(a_k)h_k \end{bmatrix}.$$
 (6)

Since the state transition function  $f(z_k)$  in the model (5) is a nonlinear function the extended Kalman filter (EKF) needs to be used to estimate the augmented states. The development of the EKF consists of two steps: linearizing the augmented model (5) and applying the standard Kalman filter to the linearized

model. The basic concept of linearization is to form the Taylor approximation of the nonlinear transition function. The resulting linear model has the next form

$$\begin{cases} z_{k+1} = F_k z_k + u_k \\ y_k = [0 \quad X_k] z_k + w_k \end{cases}$$
(7)

where

$$F_{k} = \begin{pmatrix} \frac{da_{k}}{dz_{k}} \\ \frac{dA(a_{k})h_{k}}{dz_{k}^{T}} \end{pmatrix} = \begin{pmatrix} I & 0 \\ \frac{dA(a_{k})}{da_{k}^{T}}h_{k} & A(a_{k}) \end{pmatrix}.$$
 (8)

The algorithm of EKF for this case presented as follows:

1. Prediction (before receiving OFDM symbol):

$$\hat{z}_{k|k-1} = f(z_k) = \begin{bmatrix} \hat{a}_{k-1} \\ \hat{A}_{k-1} \hat{h}_{k-1} \end{bmatrix}$$
(9)

$$P_{k|k-1} = F_{k-1}P_{k-1}F_{k-1}^H + Q_u \tag{10}$$

where  $Q_u = \begin{bmatrix} Q_{\varepsilon} & 0 \\ 0 & Q_{v} \end{bmatrix}$  – the covariance of noises  $\begin{bmatrix} \varepsilon^T & v^T \end{bmatrix}^T$ .

2. Correction (once the reception of the OFDM symbol has completed)

$$K_{k} = P_{k|k-1} \begin{bmatrix} 0\\ X_{k}^{H} \end{bmatrix} ([0 \quad X_{k}] P_{k|k-1} [0 \quad X_{k}]^{H} + Q_{w})^{-1}$$
(11)

$$\hat{z}_k = \hat{z}_{k|k-1} + K_k(y_k - [0 \quad X_k]\hat{z}_{k|k-1})$$
(12)

$$P_k = P_{k|k-1} - K_k [0 \quad X_k] P_{k|k-1}$$
(13)

where  $Q_w$  – the covariance of noise  $w^T$ .

Thus, after the linearization of state space model (5), the extended Kalman filter can be applied to estimate the channel frequency response on the pilot subcarriers and interpolation in time domain that should improve the quality of communication.

## References

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