## Improving the accuracy of the SINS for a fast-rotating, controlled object by measuring the apparent acceleration

A method for increasing the accuracy of determining the orientation in a strapdown inertial navigation system for a fast-rotating object around the longitudinal axis is proposed. The method is based on the use of measurements of the apparent acceleration vector from specially arranged accelerometers for periodic correction of the roll angle.

The problem to be solved.
The use of strapdown inertial navigation system (SINS) for a highly dynamic object is problematic because of the strong influence on the accuracy of the system of the sensors' multiplicative errors. One of the possible ways to improve the accuracy of the SINS is to correct the state vector periodically using information from satellite radio navigation systems. However, the disadvantages of this method are, firstly, the dependence on external conditions, and secondly, a small frequency of implementation of the correction, due to the low frequency of updating the satellite information. All this makes it problematic to use such a method in conditions of highly dynamic objects. So it is necessary to develop new methods of correction in these conditions.

The essence of the solution.
An algorithm for calculating with high accuracy orientation of a fast rotating object around the longitudinal axis is proposed by the correcting of the roll angle from measurements of the apparent acceleration vector by means of specially arranged accelerometers. Due to rotation, such measurements have a modulated character and moments of passage through a maximum or minimum. Also they carry information about the exact orientation of the object on the roll.

Explanation and justification of the decision.
Let the object, in addition to the trajectory movement, also perform a rotation around its longitudinal axis x , co-directional with the velocity vector, with a high speed $\omega$. Let's call the angle of rotation about the axis $x$ a roll angle $\gamma$.

In SINS that is located on the object, among the standard set of meters additionally placed two accelerometers Ay and Az at a distance $\rho$ from the axis of rotation (Fig. 1).


Fig. 1 The placement of additional accelerometers on the object


Fig. 2 The qualitative character of the measurements of the $y$-accelerometer

In the method described above, with the help of three gyroscopes, three accelerometers and a receiver of satellite signals, the angles, velocities and coordinates of the object are determined. But in conditions of fast-rotation object, the error of the scale factor of the x -gyroscope leads to a rapid monotonous increase of the error in determining the roll angle. In these conditions, to increase the accuracy of the roll angle determination, it is proposed with a fourfold increase in the object rotation speed to correct its value from the readings of the $y$ - and $z$ accelerometers. Which in the general case have the form of a curve with a slow amplitude modulation and a slow trend of the midline. With the help of electronic devices, the passage through the sequence of measurements of the y-accelerometer of its local maximum or local minimum is fixed. Then, a correction signal corresponding to the true roll angle $\gamma_{\mathrm{e}}=0$ or $\gamma_{\mathrm{e}}=\pi$ is formed. For the maximum and minimum values of the $z$-accelerometer measurements, a correction signal that corresponds to the true value of the roll angle $\gamma_{\mathrm{e}}=3 / 2 \pi$ or $\gamma_{\mathrm{e}}=1 / 2 \pi$ is generated. The formed estimate of the true value of the roll angle is fed into the algorithm for determining the orientation of the SINS. Then the current roll angle is redefined by the rule $\gamma=\gamma_{\mathrm{e}}$ there.

To justify the above method, let's turn to the corresponding mathematical models.

The projection of the apparent acceleration vector of the object on the axis of sensitivity of the $y$-accelerometer has the form:

$$
\begin{equation*}
\mathrm{a}_{\mathrm{y}}(\mathrm{t})=\mathrm{W}_{\mathrm{Oy}}(\mathrm{t})-\rho \cdot \omega^{2}(\mathrm{t})+\mathrm{g} \cdot \cos \theta(\mathrm{t}) \cdot \cos \gamma(\mathrm{t}) \tag{1}
\end{equation*}
$$

where $a_{y}(t)$ - measurements of the $y$-accelerometer at time $t ; W_{O y}(t)-$ current value of the projection of the actual acceleration of the origin of the coupled system onto the axis $\mathrm{Oy} ; \rho$ - the distance of placement of the sensor of the y accelerometer from the axis of rotation $\mathrm{Ox} ; \omega(\mathrm{t})$ - the current rotation speed of the object around the longitudinal axis; $g$ - acceleration value of gravity; $\theta(\mathrm{t})$ - current pitch angle; $\gamma(\mathrm{t})=\omega(\mathrm{t}) \cdot \mathrm{t}-$ current roll angle.

Note that, $\omega(\mathrm{t})$ and $\theta(\mathrm{t})$ change in time with a speed much less than other variables.

The actual acceleration of the point $O$ is the result of the action of all external forces that are brought to the point $O$ such as gravity $\mathrm{Q}_{\mathrm{W}}=\mathrm{g} \cdot \mathrm{m}$, resistance forces of the medium $Q_{R}(t)$, lift (force) $\mathrm{Q}_{\mathrm{U}}(\mathrm{t})=\mathrm{q}_{\mathrm{U}}(\mathrm{t}) \cdot \mathrm{m}$ and, possibly, jet engine power $Q_{T}(t)($ Fig.3).


Fig. 3 The diagram of forces that are applied to the object
Since in the case of the collinearity of the velocity vector and the longitudinal axis of the object, the resistance force and the reactive force are orthogonal to the axes of the sensitivity of the $y$-accelerometer, the projection $W_{\mathrm{Oy}}(\mathrm{t})$ taking into account the rotation of the object around the Ox axis looks like

$$
\begin{equation*}
\mathrm{W}_{\mathrm{Oy}}(\mathrm{t})=\left(\mathrm{q}_{\mathrm{U}}(\mathrm{t})-\mathrm{g} \cdot \cos \theta(\mathrm{t})\right) \cdot \cos \gamma(\mathrm{t}) . \tag{2}
\end{equation*}
$$

The substitution of (2) into (1) yields

$$
\begin{equation*}
\mathrm{a}_{\mathrm{y}}(\mathrm{t})=\mathrm{q}_{\mathrm{U}}(\mathrm{t}) \cdot \cos \gamma(\mathrm{t})-\rho \cdot \omega^{2}(\mathrm{t}) . \tag{3}
\end{equation*}
$$

Thus, the measurements of the $y$-accelerometer have a slowly varying component associated with a change in the rotation speed $\omega(\mathrm{t})$. And a highfrequency component associated with a rapid change in the roll angle and modulated by a slowly varying acceleration from the lift force. $q_{U}(t)$. It follows from (3) that, for a quasistationary character $\mathrm{q}_{\mathrm{U}}(\mathrm{t})$, the local maximum on the turnover period of the object corresponds $\gamma=0$. And the local minimum is reached at $\gamma=\pi$, which allows using these values to correct the orientation of the object and confirms the presented method of such correction.

Similar arguments hold for the z-accelerometer too.
Example of realization.
Let's simulate the movement of an object that rotates about a longitudinal axis with an angular velocity of $100 \mathrm{r} / \mathrm{s}$. The error in the scale factor of the gyro is set at $0.1 \%$, the information update rate is 10 kHz . Let's build the dependence of the error of determining the roll angle with and without correction (Fig. 4).


Fig. 4 The error in determining the roll angle by algorithms with and without correction

A straight line with a slope is the error in determining the roll angle for one second according to an algorithm without correction. The second line, which is almost parallel to the abscissa axis, is a failure to determine the angle of heel according to the correction algorithm, which is performed on the extremes of the measurements of the y and z accelerometers four times during the period of rotation. On an enlarged scale, in the shortened time interval, it has the form shown in Fig. 5. The jumps in the graph correspond to the correction moments, and the slow growth with the accumulation of the error between the correction moments.

It can be seen from the graphs that the developed method significantly limits the error in determining the roll angle. The average level of residual error is related to the rotation speed and the period of the measurement update and makes the angle to which the object rotates during half of the information update period.

Thus, the above method by using accelerometers provides an increase in the accuracy of determining the orientation parameters for fast-rotating objects in comparison with known methods.

## Conclusions:

A method to increase the accuracy of the orientation calculated in the SINS for a fast-rotating object around the longitudinal axis is developed. It is based on the use of measurements of specially arranged accelerometers for periodic correction of the roll angle. The developed method significantly limits the error in determining the roll angle, and the error itself does not increase with time.

## References

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