Dynamic analysis of the lever mechanism

The iterative method of the law of movement of an initial link of the lever mechanism in time of settled regime definition, not connected with dynamic synthesis, is offered.

Introduction. As it is known from the educational literature (see the list of referred sources), for the decision of a problem of the dynamic analysis of the mechanism, it is necessary to know parameters of its dynamic model (the angular speed, the resulted moment of inertia, change of kinetic energy) at least in one position defined in the generalized coordinate. Traditional methods [1, 3, 4] suggest to use as such position, where angular speed of a link of reduction has extreme value set by the coefficient of non-uniformity of movement. But maintenance of the set coefficient of non-uniformity of movement is a condition of dynamic synthesis of the mechanism, which is not always necessary. Thus, it appears that without dynamic synthesis, when this factor is not known also it only it is necessary to define, the dynamic analysis is impracticable. Possibility of definition of the law of movement of an initial link in this case opens the nonconventional approach to the decision of a problem of dynamic synthesis [2].

Objects and problems. Let the resulted moment of inertia of the lever mechanism will be presented as follows:

\[ J(\varphi) = J_{\text{const}} + J_{\text{var}}(\varphi), \]

where \( J_{\text{const}} \) – the constant component allocated in such a manner, that a variable component \( J_{\text{var}}(\varphi) \) has the minimum 0; \( \varphi \) – the generalized coordinate.

In figure 1 the curve of energy-mass (Vittenbauer’s diagram) received in the traditional way in system of coordinates \( JO\Delta A \), expressing conformity of values of total work \( \Delta A(\varphi) \) to values of the resulted moment of inertia of the mechanism \( J(\varphi) \) is represented. After an exception of a constant component of the resulted moment of inertia \( J_{\text{const}} \), having passed to new system of coordinates \( J_{\text{var}}O_1 \Delta E \), it is possible to write down:

\[ J_{\text{var}}(\varphi) = J(\varphi) - J_{\text{const}} \]

and

\[ \Delta E(\varphi) = \Delta A(\varphi) - \Delta A^*, \]
where \( J_{\text{const}} \) – the minimum value of the resulted moment of inertia in old system \( JO\Delta A \), \( \Delta A^* \) – ordinate corresponding to this value in the same system, \( \Delta E(\phi) \) – a variable component of change of kinetic energy of the mechanism in new system of coordinates \( J_{\text{var}}O_1\Delta E \).

![Diagram]

**Fig. 1.** To the dynamic analysis of the mechanism

Point \( O_2 \) of crossing of tangents to a curve of the energy-mass, defining unknown the maximum \( \omega_{\text{max}} \) and minimum \( \omega_{\text{min}} \) values \( \omega(\phi) \), is the beginning of system of coordinates \( JO_2E \), in which the curve of energy-mass describes dependence of full kinetic energy of the mechanism from the resulted moment of inertia \( E = E(J) \). It is possible to present this dependence as follows:
\[ E = E_0 + \Delta E(\varphi) = \frac{(J_{\text{const}} + J_{\text{var}}(\varphi)) \cdot \omega^2(\varphi)}{2}. \]  

(1)

Here \( E_0 = \frac{J_{\text{const}} \cdot \omega_0^2}{2} \) - an unknown constant component of kinetic energy, where \( \omega_0 \) - some unknown angular speed at \( J_{\text{var}}(\varphi) = 0 \) and \( \Delta E(\varphi) = 0 \). To this speed on fig. 1 there corresponds straight line \( O_2O_1 \), inclined at an angle \( \psi_0 \) to an axis of abscisses.

Thus, having \( E_0 \), from expression (1) it is possible to define current values of angular speed:

\[ \omega(\varphi) = \sqrt{\frac{2 \cdot (E_0 + \Delta E(\varphi))}{J_{\text{const}} + J_{\text{var}}(\varphi)}} = \sqrt{\frac{2 \cdot (E_0 + \Delta E(\varphi))}{J(\varphi)}} \]  

(2)

The problem of the dynamic analysis can be solved an iterative way, using the received dependence (2). The method essence is easy for understanding, having addressed to figure 1. In this case the curve of energy-mass set in system of coordinates \( JOA \), it is necessary to transfer in system of the coordinates \( JO_2E \), which beginning \( O_2 \) settles down on continuation of an axis of ordinates \( \Delta A \) in a point of intersection from the straight line corresponding to set average angular speed \( \omega_{av} \), i.e. inclined at an angle \( \psi_{av} \) to an axis of abscisses. It is required to define position of this point. The block diagram of algorithm of the decision of a problem is represented in figure 2.

Having set by the initial data \( J(\varphi), \Delta A(\varphi), \omega_{av} \) and an admissible relative deviation from size of average angular speed \( \Delta_\omega \), it is necessary to pass from system \( JOA \) in system \( J_{\text{var}}O_1 \Delta E \), as it is described above. The least value of the resulted moment of inertia \( J_{\text{min}} = J_{\text{const}} \) is thus defined. Then its greatest value \( J_{\text{max}} \), and also the greatest \( \Delta E_{\text{max}} \) and least \( \Delta E_{\text{min}} \) values of function \( \Delta E(\varphi) \) are also defined.

Position of point \( O_2 \) is defined by size of kinetic energy \( E_0 \), which it is offered to search a method consecutive approaches. It is obvious, that the point \( O_2 \) settles down on a piece \( O_2'O_2'' \), which borders \( O_2' \) and \( O_2'' \) will arrange from the top and bottom points of a curve of energy-mass on distances, that with sufficient accuracy are defined by energy sizes accordingly \( 0.5J_{\text{min}}\omega_{av}^2 \) and \( 0.5J_{\text{max}}\omega_{av}^2 \), and from an axis of abscisses of system of coordinates \( J_{\text{var}}O_1 \Delta E \) in sizes
Then the mechanism is calculated. Its extreme values and new value of average angular \( \omega_{av}' \) speed are defined. The last is compared to the set \( \omega_{av} \). If the absolute relative size of their difference exceeds the set admission \( \Delta \omega \), it is necessary to find new position of the beginning \( O_2 \) of system of coordinates \( JO_2E \), having accepted new value \( E_0 \). Thus in a case, when \( \omega_{av}' > \omega_{av} \), the point \( O_2 \) settles down on the midpoint of piece \( O_2O_2' \), differently – on the midpoint of piece \( O_2O'' \). Procedure repeats. Iterative process proceeds until the deviation of the calculated value of average angular speed of a link of reduction from a preset value will not appear within the admission.
Angular acceleration of a link of reduction can be defined traditional [3, 5] or nonconventional [2] methods.

**Conclusions.**

Some advantages of the stated method.
- Possibility of performance of the dynamic analysis without dynamic synthesis.
- Possibility of creation of simple algorithm for the machine account.
- Continuity in relation to traditional methods and basic ideas.
- Presentation owing to possibility of application of a simple graphic illustration.

**References**


