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Aeronautical engineering degrading state maximal probability determination as a proof for the hybrid-optimal functions entropy conditional optimality doctrine application

The third part of the generalization for the degrading state maximal probability determination in the framework of the hybrid-optimal functions entropy conditional optimality doctrine initiated in the preceding reports was presented in the given report. The issue will be continued with a following sequence of reports.

Introduction.
Continuing the previous research dedicated to optimal periodicity of aeronautical engineering units’ maintenance, it is an important issue to find the damaged but not failed state probability parameter maximum for the optimal periodicity determination [1-34].

State of the problem.
The optimal periodicity is considered with taking into account the dynamical characteristics of the maximum probability of a non-failure state which sometimes is a proper criterion for the optimal periodicity of the aeronautical engineering units’ maintenance [1, Chapter 15, pp. 170-172, especially Sub-Chapter 15.4, p. 172, Fig. 15.2]

Purpose of the paper.
It is to prolong the proposed approach (doctrine) likewise in [8-16] based upon the Jaynes’ principle [17-19] and subjective entropy maximum principle [7, 20-23]. It resembles [24], however in actual fact follows [8-16]. A generalization has to be done with the use of the mathematical apparatus in order to opportunely reconsider the problems of [25-34] in the framework of the discussed concept.

Problem setting.
Having determined, in a simplified system of the possible discrete states: “0” – the up state of the system; “1” – damage; “2” – failure; randomly changed in time $t$ – deemed to be a continuum [24], at the initial conditions of the states’ probabilities: $P_0|_{t=t_0} = 1$, $P_1|_{t=t_0} = P_2|_{t=t_0} = 0$, $t_0 = 0$, the wanted probabilities of $P_i$, [8-10, p. 30, (42)-(45)]:

$$P_0(t) = \frac{k_1 e^{k_1 t} - k_2 e^{k_2 t}}{k_1 - k_2} + a_1 \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} + \frac{b_1}{k_1 k_2} + \left(-\frac{b_1}{k_2(k_2-k_1)} - \frac{b_1}{k_1 k_2}\right) e^{k_1 t} + \left(\frac{b_1}{k_2(k_2-k_1)}\right) e^{k_2 t};$$

$$P_1(t) = \lambda_{01} \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} +$$

1.2.11
\[ + \frac{c_1}{k_1 k_2} + \left( -\frac{c_1}{k_2 (k_2 - k_1)} - \frac{c_1}{k_1 k_2} \right) e^{k_1 t} + \left( -\frac{c_1}{k_2 (k_2 - k_1)} \right) e^{k_2 t} \], \quad (2)

\[ P_2(t) = \frac{\lambda_{02}}{k_1 - k_2} e^{k_1 t} - e^{k_2 t} + \]

\[ + \frac{d_1}{k_1 k_2} + \left( -\frac{d_1}{k_2 (k_2 - k_1)} - \frac{d_1}{k_1 k_2} \right) e^{k_1 t} + \left( -\frac{d_1}{k_2 (k_2 - k_1)} \right) e^{k_2 t} \], \quad (3)

where

\[ k_{1,2} = -\frac{e_1 \pm \sqrt{e_1^2 - 4 f_1 g_1}}{2 f_1} , \quad e_1 = \mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10} + \lambda_{01} + \lambda_{02} \], \quad (4)

\[ f_1 = 1 , \quad g_1 = b_1 + c_1 + d_1 \], \quad (5)

\[ a_1 = \mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10} , \quad b_1 = \lambda_{12} \mu_{20} + \mu_{10} \mu_{20} + \mu_{10} \mu_{21} \], \quad (6)

\[ c_1 = \lambda_{01} \mu_{20} + \lambda_{01} \mu_{21} + \lambda_{02} \mu_{21} , \quad d_1 = \lambda_{01} \lambda_{12} + \lambda_{02} \lambda_{12} + \lambda_{02} \mu_{10} \], \quad (7)

and the corresponding values of the failure intensities \( \lambda_{ij} \) and restoration intensities \( \mu_{ji} \) for the three states system transitions; one need to go on to obtain the maximum of the non-failure state probabilities.

**Traditional concept.**

Within this report material, it is supposed to give some kind of a proof [8-10, pp. 31, 32, (46)-(54)] to the speculations (contemplations, studies, thoughts, considerations, assumptions, theories, guesswork, suppositions etc.) involving an implementation of the forthcoming hybrid-optimal functions entropy conditional optimality doctrine, prior presenting that concept’s provisions [8-16].

In order to prove the statements formulated in the following reports, let us consider the first derivative of the probability of the supposedly damaged but not failure (ruined, crash, break, fracture, split, crack, rupture) state \( P_1 \), Eq. (2), with respect to time \( t \) [8-10, pp. 31, 32, (46), (47)]:

\[ \frac{dP_1(t)}{dt} = \frac{\lambda_{01}}{k_1 - k_2} \left( k_1 e^{k_1 t} - k_2 e^{k_2 t} \right) + k_1 \left( -\frac{c_1}{k_2 (k_2 - k_1)} - \frac{c_1}{k_1 k_2} \right) e^{k_1 t} + \]

\[ + k_2 \left( -\frac{c_1}{k_2 (k_2 - k_1)} \right) e^{k_2 t} \], \quad (8)

\[ \frac{dP_1(t)}{dt} = -\frac{k_1 k_2 \lambda_{01}}{k_1 k_2 (k_2 - k_1)} \left( k_1 e^{k_1 t} - k_2 e^{k_2 t} \right) + \]

\[ + \left( -\frac{k_1 k_1 c_1}{k_1 k_2 (k_2 - k_1)} - \frac{k_1 c_1 (k_2 - k_1)}{k_1 k_2 (k_2 - k_1)} \right) e^{k_1 t} + \left( -\frac{k_1 k_2 c_1}{k_1 k_2 (k_2 - k_1)} \right) e^{k_2 t} \]. \quad (9)

Equalizing Eq. (9) to zero yields [8-10, pp. 32, (48)]:

1.2.12
\[
\frac{dP_1(t)}{dt} = -k_1k_2\lambda_01\left(k_1e^{k_1t} - k_2e^{k_2t}\right) - k_1k_1c_1e^{k_1t} - k_1\lambda_1\left(k_2 - k_1\right)e^{k_1t} + k_1k_2c_1e^{k_2t} = 0.
\]

(10)

Which means the zero value of the nominator [8-10, pp. 32, (49)]:

\[-k_1k_2\lambda_01\left(k_1e^{k_1t} - k_2e^{k_2t}\right) - k_1k_1c_1e^{k_1t} - k_1\lambda_1\left(k_2 - k_1\right)e^{k_1t} + k_1k_2c_1e^{k_2t} = 0.\]

(11)

And after a few obvious identical transformations, it yields [8-10, pp. 32, (50)-(52)]:

\[(\lambda_1k_2 + c_1)e^{k_2t} = (\lambda_01k_1 + c_1)e^{k_1t}.\]

(12)

From (12) [8-10, pp. 32, (53)]:

\[e^{(k_2-k_1)t} = \frac{\lambda_01k_2 + c_1}{\lambda_01k_1 + c_1}, \quad (k_2-k_1)t = \ln\left(\frac{\lambda_01k_2 + c_1}{\lambda_01k_1 + c_1}\right).\]

(13)

Finally the optimal solution, maintenance periodicity, expressed with [8-10, pp. 32, (54)]:

\[t^*_p = \frac{\ln(\lambda_01k_1 + c_1) - \ln(\lambda_01k_2 + c_1)}{k_2 - k_1}.\]

(14)

The consideration for the probabilities extremums and further generalization steps in the following (1)-(14) ideas will appear in the next report.

References


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