Investigation of the influence of teeth correction of worm gear with involute worm on contact parameters, wear and life

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Abstract. The paper presents a method for determining the effect of teeth correction in an involute worm gear on the contact strength, wear and life of teeth of the worm wheel. As a result of the teeth correction, the profile of the worm wheel teeth undergoes displacement (positive or negative) with respect to the initial profile (uncorrected) and formation of a tooth profile by some other section of the involute. As a result, the contact pressures and teeth wear decrease while the gear life increases. The regularities regarding the effect of correction on contact and tribocontact parameters are established.

It has been found that, in contrast to the uncorrected gear, when the correction coefficient $x_2$ is positive, the maximum contact pressures decrease; when the correction coefficient has a negative value $-x_2$, the pressures increase. The radii of teeth curvature increase with an increase in $x_2$ and they decrease with a decrease in $x_2$. The wear of the worm wheel teeth decreases at $x_2 > 0$ and increases at $x_2 < 0$ compared to the case when $x_2 = 0$. The life of the gear increases at $x_2 > 0$ while at $x_2 < 0$ it decreases.

1. Introduction

Archimedes worm gears are widely used in various devices and equipment. On meshing a sliding friction is generated, which leads to the wear of teeth of the worm wheel [1]. For this reason, it is required that the life of such a gear and the wear of its wheel be determined already at the stage of design. The literature reports methods for determining abrasive wear of worm gear teeth [2 - 7]; however, the proposed methods cannot be used for uncorrected and corrected gears. Accordingly, in [2,3] the effect of loading on the contact parameters at EHDL was investigated. The studies [4, 5] report the results regarding determination of teeth wear in an uncorrected gear according to a modified Archard law which takes into account the changes in contact pressures and oil film thickness in the contact zone based on elastohydrodynamic lubrication theory. The behavior of worm gears during loading, including the occurrence of contact pressures, was investigated in [8]. In [9], a model for predicting tooth wear along the contact line is presented. The model takes account of lubricant thickness, which is also based on Archard’s law of abrasive wear. A method [10, 11] was developed for determination of contact parameters in uncorrected worm gears; later on, it was generalized and applied to corrected worm gears.
2. Research method

2.1. Unit linear wear function

According to [10, 11] the function of liner wear of teeth of a worm gear per one revolution is calculated from the formula

\[ h'_{lj} = \frac{\gamma_j \left( p'_{ljmax} \right)^{m_j}}{C_2 \left( \tau_{es} \right)^{m_j}}, \]

where

- \( t'_j = 2b_j / v_j \) is the time of contact between the meshing components at \( j \)-th point on the friction path with a length of \( 2b_j \);
- \( v_j \) is the velocity of slide at \( j \)-th point of meshing at the height of worm coils;
- \( f \) is the sliding friction coefficient;
- \( C_2, m_2 \) are the indicators of wear resistance of material of the worm wheel in a selected pair and the conditions of wear as determined in the experimental tests;
- \( \tau_{es} \approx 0,35R_{m2} \) is the temporary shear strength of material the worm wheel;
- \( R_{m2} \) is the temporary tensile strength of material of the worm wheel;
- \( 2b_j^{(w)} = 2.256 \sqrt{\frac{\Theta N'p_{i2} / bw}{}} \) is the width of contact area;
- \( p_{jmax}^{(w)} = 0.564 \sqrt{\frac{\Theta ' p_{i2}b}{w}} \) are the maximum contact pressured determined according to the Hertz formula depending on the number of meshing pairs \( w \) of teeth of the worm wheel;

\( w \) is the number of engaged tooth pairs;
\( \Theta = \left( 1-\mu_{1}^{2} \right) / E_1 + \left( 1-\mu_{2}^{2} \right) / E_2 \) is the Kirchhoff modulus;
\( \mu, E \) are the Poisson ratio and Young modulus of materials of the worm wheel, respectively;
\( b \) is the worm wheel width;
\( \rho_{ij} \) is the radius of curvature of teeth of the worm wheel at \( j \)-th point of meshing

\[ \rho_{ij} = \frac{\rho_{i2j}}{\rho_{i1} + \rho_{2j}}. \]

2.2. Curvature radius and gear geometry

Curvature radius both of involute worm thread profiles \( \rho_{1j} \) and of the worm wheel teeth \( \rho_{2j} \)

\[ \rho_{1j} = -\frac{\eta_j \tan \alpha_{cj}}{\cos^3 \alpha_{pj} \tan \gamma_{b} \cos^2 \left( \alpha_{cj} + \varepsilon_j \right)}, \quad \rho_{2j} = \frac{\rho_{1j} \rho_{2j} \sin \alpha_{pj} + \rho_{1j} e_{p\Delta_j} - e_p^2}{r_2 \sin \alpha_{pj} + \rho_{1j} - e_{p\Delta_j}}; \]

\[ x_A = x_B, x_A = r_{f1} + 0,2m, \quad x_B = r_{u1}, \quad \tan f_1 = 0,5 \left( d_1 - 2h_{f1} \right), \quad h_{f1} = 1,2m \text{ (where } \gamma \leq 15^\circ), \]
\[ h_{f1} = 1,2m_n \text{ (where } \gamma \leq 15^\circ), \quad h_{u1} = m_n \text{ (where } \gamma \leq 15^\circ), \quad r_2 = 0,5z_2m, \quad r_2 = 0,5d_2, \quad z_2 = u_2; \quad \eta_j = 0,5d_1 \cos \alpha_{cj}, \quad \tan \alpha_{cj} = \tan \alpha_{n} / \sin \gamma, \quad q = 2 \left( 1 + \sqrt{z_2} \right); \quad \alpha_n = \alpha = 20^\circ; \quad \alpha_{cj} = \arctg \frac{\sqrt{x^2 - r_b^2}}{r_b}; \]
\[ \alpha_{pxj} = \arctg \left( -\tan y_b \frac{\sqrt{x^2 - r_b^2}}{x} \right); \quad \tan y_b = \frac{m_z}{d_1 \cos \alpha_c}; \quad \varepsilon_j = \frac{180}{\pi} \frac{\sqrt{x^2 - r_b^2}}{r_b}; \]

\[ e_{pj} = \frac{r_1 - x}{\sin \alpha_{pxj}}, \quad r_1 = 0.5d_1, \quad b = 2m\sqrt{q + 1}, \]

where
- \( r_{f1} \) is the radius of a circle of worm cavity;
- \( d_1 \) is the reference diameter of the worm;
- \( h_{f1} \) is the height of worm thread base;
- \( m \) is the axial modulus of meshing;
- \( m_n = m \cos \gamma \) is the normal modulus of meshing;
- \( \gamma \) is the angle of elevation of the screw line of worm coils;
- \( z_1 \) is the number of worm coils;
- \( q \) is the diametral quotient of the worm gear;
- \( r_{o1} \) is the radius of a circle of worm coil prongs;
- \( h_{o1} \) is the height of head of the worm coil;
- \( d_2 \) is the reference diameter of the worm wheel;
- \( z_2 \) is the number of teeth in the worm wheel;
- \( u \) is the gear ratio;
- \( n_p \) is the radius of a basic circle of worm coils;
- \( \alpha_c \) is the face pressure angle;
- \( \alpha_n = \alpha = 20^\circ \) is the angle of meshing;
- \( \alpha_{cj} \) is the face pressure angle of \( j \)-th point;
- \( \varepsilon \) is an angular coordinate in every pitch (degrees);
- \( e_{pA} \) is the distance of \( j \) point from the contact point.

The section of meshing \([x_A, x_B]\) must be divided proportionately into smaller sections:
\[ x_A = j_A = j_1, \quad x_2 = j_2, \quad x_3 = j_3, \quad \ldots, \quad x_B = j_n = j_B. \]

2.3. **Slipping velocity**

The slipping velocity \( v_j \) is determined according to the formula:

\[ v_j = \frac{\omega_i x}{\cos \gamma_A}, \quad (3) \]

where \( \tan \gamma_A = m_z / 2x \); \( \omega_i = \pi n_i / 30 \) is the angular velocity of the worm; \( n_i \) is the number of revolutions of the worm.

2.4. **Wear and life of the worm gear**

The wear of the worm wheel teeth within one hour of gear operation is [10]:

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\[ \bar{h}_{2j} = 60n_2 h'_{2j}, \quad n_2 = n_1 / u, \tag{4} \]

where \( n_2 \) is the number of revolutions of the worm wheel per minute; \( h'_{2j}, \quad n_2h'_{2j} \) are the linear wear of the worm wheel teeth within one revolution and one minute of operation, respectively.

The life of the worm gear for the acceptable wear \( h_{2s} \) of the worm wheel teeth is calculated according to the formula:

\[ t_s = \left( h_{2s} / \bar{h}_{2j} \right) \tag{5} \]

2.5. Meshing force

The meshing force is calculated according to the formula [10]

\[ N' = \frac{2T}{d_1 \cos \alpha_{psj} \sin (\gamma + \rho')}, \tag{6} \]

where \( T = 9550 \cdot 10^3 (N / n_1) \) is the torque transmitted by the worm; \( \rho' = \arctg \left( \frac{f}{\cos \alpha} \right) \) is the friction angle; \( N \) is the transmitted power.

2.6. Worm wheel teeth correction

As a result of the teeth correction, the profile of the worm wheel teeth undergoes displacement (positive or negative) with respect to the initial profile (uncorrected) and formation of a tooth profile by some other section of the involute. As a result, the contact pressures and teeth wear decrease while the gear life increases. In worm gears, we can only apply correction to the worm wheel teeth. Hence, the applied displacement of the gear-cutting hob is:

\[ \xi = x_2 m, \]

where \( x_2 \leq \pm 1 \) is the correction correlation.

The interaxial distance is:

\[ a_{wk} = a_w + x_2 m, \tag{7} \]

where \( a_w = n_1 + r_2 \) is the interaxial distance in the uncorrected gear.

The reference diameter of the worm in the uncorrected gear

\[ d_{w1} = d_1 + x_2 m. \]

Therefore, the distance \( e_{psj} \) between the \( j \)-th point of contact and the point of contact in [11] will be:

\[ e_{psj} = \frac{r_{w1} - x}{\sin \alpha_{psj}}, \quad r_{w1} = d_{w1} / 2. \]

For the uncorrected worm gear, the force in meshing

\[ N' = \frac{2T}{d_{w1} \cos \alpha_{psj} \sin (\gamma + \rho')} . \]

3. Solution

Other geometrical parameters are determined according to the formulae for the uncorrected worm gear. The computations were performed using the following set of data: \( N = 3.5 \) kW, \( n_1 = 1410 \) rev/min, \( m = 6 \) mm, \( z_1 = 2 \), \( u = 25.5 \), \( f = 0.05 \), \( q = 8 \); worm-hardened steel grade 45 (HRC 50) described by \( E_1 = 2.1 \cdot 10^5 \) MPa, \( \mu_1 = 0.3 \); worm wheel ring – bronze CuSn6Zn6Pb6 described by \( E_2 = 1.1 \cdot 10^5 \) MPa, \( \mu_2 = 0.34 \); \( C_2 = 7.6 \cdot 10^6 \), \( m_2 = 0.88 \); \( \tau_{c2} = 75 \) MPa; for \( j = 1; 2; 3; 4 \) and 5, respectively \( x = 18; 20; 22; 24 \) and 26 mm; \( h_{2s} = 0.5 \) mm; with double-pair meshing.
The results of the numerical solution are given in Figures 1–6. Figure 1 shows the relationships between the maximum contact pressures $p_{\text{max}}$ and the displacement coefficient $x_2$ on entering the mesh ($j = 1$) and on leaving the mesh ($j = 5$). It was found that the positive correction of the worm wheel teeth leads to reduced pressures, while the negative correction results in their increase compared to the observations made regarding the uncorrected gear. This results in an increase in the teeth curvature summary radius $\rho_z = \rho$ at their acceptable wear $h_2^* = 0.5$ mm when $x_2$ is positive and it decrease at negative values (Figure 2).

The effect of teeth correction of the linear wear $h_2$ of the worm wheel teeth during one hour of gear operation is illustrated in Figure 3. Figure 4 shows the minimal life $t_{\text{min}}$ of the corrected gear ($j = 1$; $x = 18$ mm) at the acceptable wear $h_2^* = 0.3$ to 0.5 mm.

As a result of the applied teeth correction, when $x_2 > 0$, $\rho_z$ increases and so the wear $h_2$ (Figure 3) decreases and the gear life $t_{\text{min}}$ increases (Figure 4); when $x_2 < 0$, $\rho_z$ decreases, which causes consequences which are undesired from a practical point of view. It must be stressed that the linear wear of teeth on entering the mesh ($j = 1$) and on leaving the mesh ($j = 5$) is similar (Figure 3), although the pressures $p_{\text{max}}$ (see Figure 1) differ to a significant degree.
Figure 3. Gear teeth wear versus \(x_2\): □ – value \(\bar{h}_{2\text{max}}\) by \(j = 1\); △ – value \(\bar{h}_{2\text{min}}\) by \(j = 5\).

Figure 4. The influence of correction on gear life \(t_{\text{min}}\): △ – value \(t_{\text{min}}\) by \(h_{2*} = 0.3\) mm; □ – value \(t_{\text{min}}\) by \(h_{2*} = 0.5\) mm.

The applied teeth correction does not affect the sliding velocity \(v_j\) in meshing and its value only depends on the position of contact point at the height of the tooth (Figure 5).

Figure 5. Change in sliding rate over tooth height

We also investigated the contact area width \(2b_j\) at meshing points \(j = 1, 2, \ldots, 5\) (Figure 6a) and the time \(t'_{j}\) of unit contact. It was found that the unit sliding friction path \(2b\) decreases significantly from \(j = 1\) to \(j = 5\), so the life of the gear decreases, too. When the gear teeth enter the mesh, the parameter \(2b_1\) remains practically unchanged, but when the teeth leave the mesh the value of \(2b_5\) (bottom curve) slightly changes (Figure 6b).
Figure 6. The influence of correction on contact area per one interaction of teeth and worm wheel revolution: □ – $x_2 = 0$; ⃝ – 1; △ – $x_2 = -1$.

The discovered qualitative and quantitative relationships with respect to the effect of teeth correction on the load capacity, wear, life and sliding velocity of the gear are reported in the Conclusions section below.

Conclusions
1. It has been found that, in contrast to the uncorrected gear, when the correction coefficient $x_2$ is positive, the maximum contact pressures decrease; when the correction coefficient has a negative value – the pressures increase (Figure 1).
2. The radii of teeth curvature increase with an increase in $x_2$ and they decrease with a decrease in $x_2$ (Figure 2).
3. The wear of the worm wheel teeth decreases at $x_2 > 0$ and increases at $x_2 < 0$ compared to the case when $x_2 = 0$ (Figure 3).
4. The life of the gear increases at $x_2 > 0$ while at $x_2 < 0$ it decreases (Figure 4).

References