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**Modeling the delivery of goods using drones**

*In recent years, based on optimistic predictions about the broad application scenarios and huge economic benefits of future unmanned transport aircraft, many scientific research institutions and companies at home and abroad have proposed development plans for unmanned transport aircraft.*

**1. Overview**

Today, there is an acute problem of fast delivery of goods (medical supplies, special postal items, special cargo for users of various organizations, etc.) The urgency of delivery of such goods can be ensured with the help of drones. This type of transportation competes with traditional means of delivery. In large cities, it is difficult due to traffic jams, and in small towns, due to underdeveloped transport infrastructure [1]. These difficulties do not affect the delivery process and its urgency when using drones [2].

**2.Event Analysis**

In logistics, there is also the complexity of direct delivery of goods to the customer. This is the so-called 'last mile' task [3].Therefore, the problem of urgent delivery of goods can be viewed as an automated system with mass service objects with events that occur at random moments in time [4]. Such events form a stochastic sequence, usually called an event stream [5].

We will assume that the event stream associated with a request for delivery of goods by a service object satisfies the following conditions:

1) for any two non-intersecting time intervals, the probability of occurrence of any given number of events during one of them does not depend on the number of events that occur during the other;

2) the probability of occurrence of one event during an infinitesimal time interval $(t, t + \Delta t)$ is an infinitesimal value of order $\Delta t$;

3) the probability of occurrence of more than one event during the interval time interval $(t, t+ \Delta t)$ is infinitesimal of the highest order compared to $\Delta t$.

Let $P_m(t_1, t_2)$ denote the probability of occurrence of m events in the time interval $(t_1, t_2)$. Then conditions 2) and 3) will be written in the form:

$$P_1 (t, t+ \Delta t) = \lambda (t) + o (\Delta t) \qquad (1)$$

$$\sum_{k=2}^{\infty} P_k(t + \Delta t) = o(\Delta t), \qquad (2)$$

where $\lambda (t)$ is some non-negative function.

- **The equation that the event will not happen.**

The problem is to find the probability that m events (m = 0, 1, 2, ...) will appear in a given time interval (t0, t) for a stream of events satisfying conditions 1), 2) and 3).

Considering the moment t0 to be fixed, we denote the probabilities by $P_m$ (t) ($m$ = 0, 1, 2,…)

To calculate $P_0$ $(t)$, we note that $P_0$ $(t+\Delta t)$ is the probability of intersection of two events: no event in the interval $(t_0, t)$ and no event in the interval $(t, t+\Delta t)$. According to condition 1), these events are independent. Therefore:

$$P_0(t+\Delta t) = P_0(t)\, P_0(t, t+\Delta t). \qquad (3)$$

Based on (1) and (2):

$$P_0(t, t+\Delta t) = 1 - \sum_{k=1}^{\infty} p_k\,(t, t+\Delta t) = 1 - \lambda(t)\,\Delta t + o(\Delta t). \quad (4)$$

Substituting this into expression (3), we get:

$$P_0(t+\Delta t) = P_0(t) - P_0(t)\,\lambda(t)\,\Delta t + o(\Delta t),$$

Whence

$$\frac{p_0(t+\Delta t) - p_0(t)}{\Delta t} = -\lambda(t)p_0(t) + \frac{o(\Delta t)}{\Delta t}.$$

As $\Delta t \to 0$, the right-hand side of this equality tends to a certain limit, $\lambda(t)\, P_0(t)$. Consequently, there is a limit to the left-hand side. Thus, the probability $P_0(t)$ is differentiable for any t, and in the limit at $\Delta t \to 0$ we obtain the differential equation:

$$P`_0(t) = -\lambda(t)\, P_0(t). \qquad (5)$$

To find the initial value of the probability $P_0$ (t), it is enough to set t = $t_0$ in (4) and move to the limit when $\Delta t \to 0$. Then we get $P_0(t_0) = 1$.

- **Equations for the probabilities of different numbers of events.**

To derive the equations for the probabilities P1 (t), P2 (t), ...note that m events can appear in the time interval (t0, t+ Δt) in one of the following m+1 incompatible ways: all m events appear in the interval (t0, t) and none in the interval (t, t+ Δt),

m-1 events appear in the interval (t0, t) and one in the interval (t, t+ Δt), etc., all m events appear in the interval (t, t+ Δt). Therefore, based on the axiom of probability addition and the theorem of multiplication of probabilities of independent events P(A1A2...An)=P(A1)P(A2)...P(An), we have:

$$Pm(t+\Delta t) = Pm(t)\, P0(t, t+\Delta t) + Pm\text{-}1(t)\, P1(t, t+\Delta t) + \ldots + P0(t)Pm(t, t+\Delta t).$$

Hence, in accordance with (1), (2), (4), we obtain:

$$Pm(t+\Delta t) = Pm(t) + [Pm\text{-}1(t) - Pm(t)]\,\lambda(t)\,\Delta t + o(\Delta t).$$

So,

$$\frac{P_m(t+\Delta t) - p_m(t)}{\Delta t} = \lambda(t)[P_{m-1}(t) - P_m(t)] + \frac{o(\Delta t)}{\Delta t}$$

Reasoning further in exactly the same way as in the derivation of equation (5), we obtain the differential equation

$$P`m(t) = \lambda(t)\,[Pm\text{-}1(t) - Pm(t)] \ (m = 1, 2, \ldots). \qquad (6)$$

The initial probability values P1 (t), P2 (t), ... are all zero because P0 (t0) = 1, Pm (t0) = 0 (m = 1, 2, ...).

- **Solving equations.**

Taking the following as the independent variable

$$\mu = \int_{t_0}^{t} \lambda(\tau) d\tau, \qquad (7)$$

Let's rewrite equations (5) and (6) as follows

$$\frac{dP_0}{d_\mu} = -P_0, \ \frac{dP_m}{d_\mu} = -P_m + P_{m-1} \ (\mathrm{m} = 1, 2, \ldots) \qquad (8)$$

The initial conditions will be P0 = 1, Pm = 0, (m = 1, 2, ...) with μ=0. It is easy to verify by direct substitution that the integrals of equations (8) satisfying the initial conditions are defined by the formula:

$$P_m = \frac{\mu^m}{m!} e^{-\mu} \qquad (\mathrm{m} = 0, 1, 2, \ldots) \qquad (9)$$

Thus, for a given time interval (t0, t), we have an even set of elementary events: no event in this interval, one, two, etc., and the probabilities of these events are determined by formula (9). Thus, formula (9) defines the probability distribution. Therefore, a stream of events satisfying conditions 1), 2) and 3) is called a Poisson stream. The parameter μ of the Poisson distribution represents the average number of events occurring in a given time interval (t0, t). The function μ(t) is called the intensity of the Poisson flow.

- **EXAMPLE. Estimating the probability of receiving requests for delivery of goods**

There arem=100 service points (SPs) in the service area that can submit a service request. The probability that within t - minutes the service point will contact you is equal to Pз=0.01.

It is necessary to estimate the probability that within t - minutes they will contact you

three software programs

less than three LAs;

more than three LAs;

at least one LA.

According to the condition m = 100, Pz = 0.01, and given that the occurrences of software are independent values when their number is large and the probability of this event is low (Pz = 0.01), we can use the Poisson formula:

$$P_m = \frac{\mu^m}{m!} e^{-\mu} \qquad (\mathrm{m} = 0, 1, 2, \ldots),$$

where μ is the parameter of Poisson's law.

In this case, the value of X is distributed according to the Poisson law and its probability takes the value m.

Find the value of the parameter μ:

μ = m · Pз = 100 · 0,01 = 1.

The probability that three software programs will communicate simultaneously (m = 3):

$$P_m(3) = \frac{e^{-1}}{3!} = \frac{0{,}367879}{6} = 0{,}0613$$

Find the probability that less than three software programs will communicate:

P (<3) = P100 (0) + P100 (1) + Pm (2) = e-1 + e-1 + $\frac{e^{-1}}{2} = \frac{5}{2} e^{-1} = \frac{5}{2} \cdot 0{,}367879 = 0{,}9197$

The probability that more than three software will communicate (P (> 3)) will have the following value.

Since the events "more than 3 software will communicate" and "no more than 3-LAs" P = P (> 3), and Q = P (< 3) - are opposite events, so P + Q = 1, i.e.

P (> 3) = 1 – P (<3) = 1 – [P100(0) + P100(1) + P100(2) + P100(3)].

Тоді P (> 3) = 1 – [0,9197 + 0,0613] = 0,019

The probability that at least one PO (we denote the probability of this event by P).

The events "at least one aircraft will communicate" and "no aircraft will communicate" (denoted by the probability of this event by Q) are opposite. Therefore, P + Q = 1.

Hence, the probability that at least one UA will get in touch is equal to:

P = 1 – Q = 1 – P100 (0) = 1 – e-1 = 1 – 0,36788 = 0,632

Other transportation problems are solved in a similar way if the flow of events can be described by Poisson's law.

**3. Equation for an event that did not occur**

Let's set the task: for a stream of events that satisfy conditions 1), 2) and 3), find the probability that m events ($m$ = 0, 1, 2, ...) will occur in this time interval $(t_0, t)$.

Considering time $t_0$ fixed, we denote the probabilities $Pm(t)$ ($m$ = 0, 1, 2, ...)

To calculate $P_0(t)$, we note that $P_0(t+\Delta t)$ is the probability of intersection of two events: no event in the interval $(t_0, t)$ and no event in the interval $(t, t+\Delta t)$. According to condition 1), these events are independent. Therefore:

$$P_0(t + \Delta t) = P_0(t)\, P_0(t, t + \Delta t). \quad (3)$$

Based on (1) and (2):

$$P_0(t, t + \Delta t) = 1 - \sum_{k=1}^{\infty} p_k\,(t, t + \Delta t) = 1 - \lambda(t)\,\Delta t + o(\Delta t) \quad (4)$$

Substituting this into expression (3), we obtain:

$$P_0(t + \Delta t) = P_0(t) - P_0(t)\,\lambda(t)\,\Delta t + o(\Delta t),$$

of which:

$$(P_0(t+\Delta t) - P_0(t))/\Delta t = -\lambda(t)\, P_0(t) + o(\Delta t)/\Delta t\,.$$

As $\Delta t \to 0$, the right-hand side of this equation tends to a certain limit - $\lambda(t)\, P_0(t)$. Consequently, the left-hand side has a limit. Thus, the probability $P_0(t)$ is differentiable at any $t$, and at the limit when $\Delta t \to 0$, we get a differential equation:

$$P`_0(t) = -\lambda(t)\, P_0(t). \quad (5)$$

To find the initial value of the probability $P_0(t)$, it is enough to substitute $t = t_0$ in (4) and take the limit $\Delta t \to 0$. Then we get $P_0(t_0) = 1$.

We have:

$$Pm(t + \Delta t) = Pm(t)\, P_0(t, t + \Delta t) + Pm.$$

And using (5) for $P_0(t)$, we obtain:

$$Pm`(t) = \lambda(t)\,[Pm\text{-}1(t) - Pm(t)],\ m \geq 1. \quad (6)$$

Thus, we have a system of differential equations (5) and (6) for determining the probabilities $Pm(t)$ ($m$ = 0, 1, 2, ...) in a stream of independent events.

To solve this system, initial conditions are required, which can be obtained from (4). For example, if $\lambda(t) = \lambda$ is a constant, then (4) can be obtained:

$$P_0(t) = exp(-\lambda(t - t_0)), \quad (7)$$

$$Pm(t) = [\lambda(t - t_0)]m\ exp(-\lambda(t - t_0)) / m!,\ m \geq 1. \quad (8)$$

Thus, from (7) and (8), we can obtain the probability that m events will appear in the interval $(t_0, t)$ at a constant value of $\lambda$.

It can be seen that depending on the value of $\lambda$ (i.e., the frequency of events), the probabilities of different numbers of events can change from one moment to the next. This model can be used to model the delivery of goods using drones, where the frequency of events (in our case, the appearance of a drone for delivery) may depend on the time of day, weather conditions, demand for delivery, etc.

This equation is derived using the Bernoulli method and reflects the probability of m events occurring within a certain time period. The equation has the form:

$$Pm(t) = [\lambda(t) / m] \int_{t_0}^{t} Pm\text{-}1(\tau)\, d\tau,$$

Where $\lambda(t)$ is the intensity of the event flow (i.e., the average number of events per unit of time), $Pm(t)$ is the probability that m events will occur in the time interval $(t_0, t)$.

This is a recursive equation that can be used to calculate the probabilities $P_0(t), P_1(t), P_2(t)$, etc. To calculate $P_0(t)$, you need to take $P_0(t) = exp(-\int_{t_0}^{t} \lambda(\tau)\, d\tau)$ with the initial condition $P_0(t_0) = 1$.

## Conclusions:

In this text, we consider the modeling of the delivery of goods using drones, and derive mathematical formulas for calculating the probabilities of events associated with this delivery. In particular, we derive a differential equation for the probability that no events will occur in the time interval from $t_0$ to t, as well as formulas for the probabilities that different numbers of events will occur in this time interval.

These formulas can be useful for predicting and optimizing the delivery of goods using drones, as well as for developing algorithms for managing this process.

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