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On the one algorithm for solving the traveling salesman problem

It is consider a technique for the sequential application of flow schemes for distributing a homogeneous resource for solving the traveling salesman problem, which is formulated as the problem of finding a route to visit a given number of cities without repetitions with a minimum duration of movement. The task of formalizing the algorithm for solving the traveling salesman problem by the method of streaming resource distribution using the backtracking scheme is solved.

Problem statement.

The traveling salesman problem (TSP) is a combinatorial problem that can be solved using mathematical programming methods. To reduce the problem to a general form, we number the cities by numbers (*1, 2, 3, ..., n*), and describe the traveling salesman's route by a cyclic permutation of numbers $t = (j_1, j_2, ..., j_n, j_1)$, where all $j_1, ..., j_n$ are different numbers [1].

The set of cities can be considered as the vertices of some graph with given distances (or travel time) between all pairs of vertices c_{ij} that form the matrix $C=(c_{ij})$, $i,j=\overline{1,n}$. We assume that the matrix is symmetric. The formal problem then is to find the shortest in time route t that goes through each city and finishes at the starting point. In this formulation, the problem is called the closed traveling salesman problem, which is a well-known mathematical integer programming problem.

Let us formulate a mathematical model of the TSP problem. Let $I=\{1,...,n\}$ be the set of vertex indices of the problem graph. The objective function is the total distance or time of the route, including all the vertices of the task graph. The parameters of the problem are the elements of the matrix $C=(c_{ij})$, $i,j \in I$.

Shift tasks are elements of the binary matrix of transitions between vertices $X=\{x_{ij}\}$, $i,j \in I$, which are equal to 1 if there is an edge (v_i, v_j) in the constructed route for the task, 0 otherwise. The shortest route in terms of time is optimal:

$$\sum_{i\in I} \sum_{j\in I, j\neq i} c_{ij} x_{ij} \to \min \qquad (1)$$

with constraints

$$\sum_{j\in I, j\neq i} x_{ij} = 1,\ i \in I, \qquad \sum_{i\in I, j\neq i} x_{ij} = 1,\ j \in I, \qquad (2)$$

$$v_i - v_j + n x_{ij} \le n-1,\ 1 \le i \neq j \le n.$$

The last inequality ensures the connectivity of the vertex traversal route; it cannot consist of two or more unconnected parts.

Algorithms that allow solving the problem of finding the optimal route are divided into exact and heuristic. Exact methods guarantee finding the optimal solution to the problem in a certain time or taking into account certain resource constraints. In this case, the search for solutions is based on optimization methods such as linear programming, dynamic programming, or the branch and bound method [2].

There is consider a method for formalizing the algorithm for solving the traveling salesman problem using the method of streaming resource allocation [3] and using the backtracking scheme [4]. The use of Orlin's method to optimize the flow distribution on the graph is proposed. The scheme of formalization of the procedure for using the method with the implementation of the backtracking scheme for solving the traveling salesman problem with the minimum duration of movement along the route is briefly described. A variant of accelerating the speed of the developed algorithm is proposed, which consists in using a greedy technique in the procedure for selecting route sections: the planning of each next stage of movement is determined based on the choice of the fastest direction of movement, which makes it possible to obtain a constructive scheme for solving the traveling salesman problem.

The greedy method proposed by the authors assumes consideration at each stage of the formation of the route of the fastest in time section of the route of movement. A combined approach based on the method of resource allocation and greedy choice of the direction of movement made it possible to implement a constructive scheme for solving the traveling salesman problem, that can be formulated as the following *recursive algorithm* for a network of *N* nodes and a given travel time for each pair of vertices:

Step 0. We form the initial information for the flow distribution method. The starting vertex of the traveling salesman route defines a subset of the initial nodes of the method, the set of directions from it defines a subset of intermediate nodes, and the graph vertices accessible from this subset defines the set of end nodes.

Based on the Orlin method of flow distribution, we determine the time to reach each of the end vertices on a subnet of initial, intermediate, and final vertices.

We select the shortest travel time and the corresponding stage of the route, mark the selected vertices and proceed to the formation of data for a new flow distribution problem. We pass to the next step of the algorithm.

Step s, s=1,2,… We construct new subsets of initial, intermediate, and final vertices, excluding from further consideration the previously noted vertices.

If at the current step it is impossible to determine new subsets (all vertices are marked), we return to the previous step, unmark the route stage, marking the dead end direction, and move on to the next possible one by choosing the fastest direction of movement.

We repeat this process until we reach the end point of the route, which coincides with the starting point.

If the route is built, but does not include all the vertices of the graph, we return to the previous levels and rebuild all the working subsets, choosing new directions of movement, taking into account the speed of movement.

Final step. As a result of the work, we finally obtain a cyclic permutation of the numbers of the vertices of the graph, which determines the sequence of stages of the traveling salesman's route.

Experiments

To analyze the efficiency of the algorithm, computational experiments were carried out, in which various methods (complete search, greedy, annealing and the one proposed above) were used to solve the traveling salesman problem on a network of 11 points. The graph of the network of movements with the given time costs is shown in Figure 1.

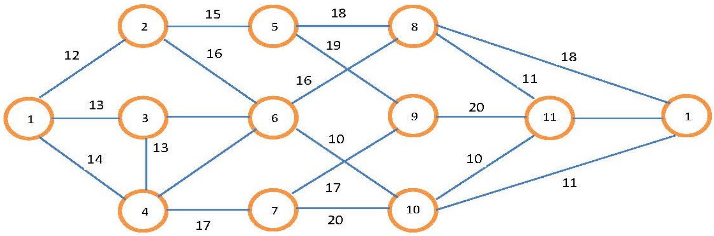

Fig. 1. The network and the given travel time for each pair of points for the TSP

Results

The results of the numerical experiments performed are shown in Table 1.

The optimal route in the considered problem was found by full search and is determined by the sequence of numbers 1, 2, 5, 9, 7, 4, 3, 6, 8, 11, 10, 1 or 1, 2, 5, 9, 7, 4, 3, 6, 10, 11, 8, 1. The proposed algorithm made it possible to quickly find a route for visiting all graph vertices (1, 4, 3, 6, 2, 5, 9, 7, 10, 11, 8, 1), but the time it took more to move along this route.

As a result of the computational experiments the efficiency of using the developed algorithm was established, the obtained solutions are compared with the solutions found by other exact and heuristic methods.

Table 1.

The comparison of search time and solutions of the TSP for N=11

Calculation method	Operation time	Optimal solution	Solution characteristic
Complete search	30 sec	157 h	Exact
Greedy algorithm	21 sec	169 h	Approximate
Annealing method	23 sec	174 h	Approximate
The proposed algorithm	25 sec	169 h	Approximate

References

1. Schrijver A. Theory of Linear and Integer Programming. – Wiley, 1998. – 484 p.
2. Korte B., Vygen J., Combinatorial Optimization: Theory and Algorithms (Algorithms and Combinatorics). - Springer Berlin, Heidelberg, 2018. - 455 p.
3. Orlin J.B. A Faster strongly polynomial minimum cost flow algorithm// Operations research. - 1993. - V. 41. – Iss.2. - P.338–350.
4. Watson Des. A Practical Approach to Compiler Construction. – Springer, 2017. – 254 p.