UDC 539.3:543.1

B.P. Maslov, Dr. Sci.
(S.P. Timoshenko Institute of Mechanics NAS Ukraine, Ukraine)

Computer modelling of laminated metal-polymer composites and long-term strength prediction

A nonlinear creep problem of the composite laminate is solved within the framework of a second-order nonlinear theory. The hereditary functionals are used to construct the general constitutive equations. Schapery's correspondence principle is applied to solve creep problem. The multiscale analysis is done using micromechanical and FEA modeling.

Introduction

Laminated composites with an aluminum and polymer constituents are essential engineering materials. Such materials are widely used in aviation and space technology [1, 2], When modeling the long-term response of composites with a polymer matrix, it is necessary to consider the effects of creep, which develops even at low enough temperatures. For this purpose, we use the following characteristics to describe the deformation and strength of composites: the creep strain, the rate of creep strain, the relaxation time, the ratio of creep limits, and durability.

One of the urgent problems of the mechanics of hereditary creep of com posite materials is to predict the averaged creep properties of the composite based on the properties of its components, their volume content, and reinforcement methods. Many theoretical and experimental papers are devoted to studying various aspects of the creep process of isotropic composite materials. An overview of the obtained results is presented in [3, 4]. The vast majority of the performed studies refer to the linear region of long-term viscoelastic deformation. All solutions are built based on the Boltzmann - Volterra linear theory of viscoelasticity [4]. It is a well-known fact that for most metal composites, the linearity range is relatively small, and satisfactory results can be obtained at low stress and only for short loading duration [2]. Nonlinear equations of viscoelasticity are given in [5]. However, the representation by infinite series of multiple integrals makes it impossible to identify integral kernels and determine their parameters in typical experiments. An approach based on the similarity of isochronous creep diagrams is more promising for building nonlinear models of hereditary creep [6]. This new algorithm is further developed due to the expansion of the initial condition of similarity, which includes the diagram of instantaneous deformation as an isochrone for the zero moment. The extension of the similarity condition made it possible to build a nonlinear creep model with a time-invariant nature of the nonlinearity, which is determined by the instantaneous deformation diagram. Within the framework of the model, the problem of determining nonlinear creep deformations of polymer with reinforcing fibers was solved [5]. Nonlinear creep deformations of fibrous unidirectional composites during stretching along the direction of reinforcement are determined. In [4], Volterra's theory of hereditary elasticity is used to solve the FEA problems of creep mechanics. The theory describes inverse processes and assumes a linear relationship between stresses and strains. Therefore, it sometimes cannot be used to describe the creep of metals, even in the first approximation [2].

State of the Problem

Deformation in hereditary materials is determined by the stress history $\boldsymbol{\sigma}(u),\ \ (u \in [0,t])$, as well as the initial conditions $\boldsymbol{\sigma}(0) = \mathbf{0}$. For a linear medium, after integration in, it is possible to write down in the general tensor form [5]

$$\mathbf{e}(\mathbf{x},t) = \int_0^t \mathbf{J}(t-u)\frac{d}{du}\boldsymbol{\sigma}(\mathbf{x},u)du, \tag{1}$$

where $\mathbf{J}(t) = \mathbf{J}^e(\boldsymbol{\sigma})g(t)$ is the nonlinear creep tensor function (retardation). In a short symbolic form [1, 6], we have

$$\mathbf{e}(\mathbf{x},t) = \frac{d}{dt}(\mathbf{J} * \boldsymbol{\sigma})(\mathbf{x},t) = (\boldsymbol{\sigma} * \dot{\mathbf{J}})(\mathbf{x},t), \tag{2}$$

where the asterisk denotes the integral convolution operation. Thus, the expression $(\boldsymbol{\sigma} * \dot{\mathbf{J}})(t) = (\boldsymbol{\sigma} * d\mathbf{J})(t)$ is a Stieltjes convolution [3]. If $f(t)$ is some continuous function on the interval $0„\ t < \infty$ and it behaves as an exponent when $t \to \infty$, then the Laplace--Carson (LC) transformation of the function reads [2]

$$LC\{f(t)\} = f(s) = s\int_0^\infty f(t)e^{-st}dt. \tag{3}$$

Applying the LC transformation to (1) and (2), we find

$$\boldsymbol{\sigma}^e(s) = (\mathbf{C}\mathbf{e})(s) = (g\ \boldsymbol{\sigma})(s). \tag{4}$$

Here it is assumed that the instantaneous elastic response of the material is physically linear. That is, there is an LC transformation $\mathbf{C}(s)$ from the relaxation function $\mathbf{C}(t)$. We also apply the law of instantaneous deformation of the second order of deformations [2, 5]

$$\boldsymbol{\sigma}(s) = \mathbf{C}(s)\mathbf{e}(s) - \boldsymbol{\beta}(s), \quad \boldsymbol{\beta}(s) = -\underline{\mathbf{C}}(s)h(s)\mathbf{e}(s),$$

where $\boldsymbol{\beta}(\mathbf{e})$ is hardening stress [2] (it is the nonlinear function of infinitesimal strain tensor $\mathbf{e}(t)$ or $\mathbf{e}(s)$ in LC domain), $h(t)$ is the reduced relaxation function [6].

In the linear theory of viscoelasticity, the solution to problems can be obtained using the correspondence principle [1, 3]. It is natural to generalize this principle to problems of hereditary creep. Here we use quasilinear variant of the hereditary creep equations, where the concept of so-called modified stresses $\boldsymbol{\sigma}^e(t) = \partial W / \partial \mathbf{e}(t)$ and modified (restored elastic) strains $\mathbf{e}^e(t) = \partial U / \partial \boldsymbol{\sigma}(t)$ is used. The function $W(\mathbf{e})$ is elastic energy, the function $U(\boldsymbol{\sigma})$ is complementary elastic energy. Following the accepted hypothesis, we assume that the material exhibits an instantaneous elastic reaction and denote by $\mathbf{e}^e(\mathbf{x},t)$, $\boldsymbol{\sigma}^e(\mathbf{x},t)$, $\mathbf{u}^e(\mathbf{x},t)$ instantaneous elastic deformation, stress, and displacement, respectively. A material with elastic properties is defined as a medium whose behavior corresponds to the first and second laws of thermodynamics. From this follows the existence of the stored energy function $W(\mathbf{e})$ and additional energy $U(\boldsymbol{\sigma})$, which make it possible to find instantaneous deformation (during creep) or instantaneous stress (relaxation process)

$$\begin{aligned} U &= U(\boldsymbol{\sigma}, \mathbf{x}, t); \quad \mathbf{e}^e(t) = \frac{\partial U}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}, \mathbf{x}, t), \\ W &= W(\mathbf{e}, \mathbf{x}, t); \quad \boldsymbol{\sigma}^e(t) = \frac{\partial W}{\partial \mathbf{e}}(\mathbf{e}, \mathbf{x}, t). \end{aligned} \tag{5}$$

If a hereditary body is initially undisturbed, mass forces $\mathbf{b}$ and tractions $\mathbf{t}$ are given, then the solution of the nonlinear problem of hereditary creep (equations, and) is as follows

$$\boldsymbol{\sigma}(t) = \boldsymbol{\sigma}^e(t), \quad \mathbf{e}(t) = (g * d\mathbf{e}^e)(t), \quad \mathbf{u}(t) = (g * d\mathbf{u}^e)(t), \tag{6}$$

where the Stieltjes convolution is defined by \eqref{ma3}. The field variables $\boldsymbol{\sigma}^e(\mathbf{x})$, $\mathbf{e}^e(\mathbf{x})$, $\mathbf{u}^e(\mathbf{x})$ satisfy the equation of the corresponding problem of the nonlinear theory of elasticity together with the same boundary conditions [1, 3]. Thus, $\mathbf{u}^e(\mathbf{x})$ satisfies the equations of the corresponding nonlinear elastic problem together with the same mass forces $\mathbf{b}^e(\mathbf{x}) = \mathbf{b}(\mathbf{x})$ in the body and the boundary conditions $\mathbf{t}^e(\mathbf{x}) = \mathbf{P}(\mathbf{x})$.

A local problem of the mechanics of hereditary creep

Aluminum alloys of several classes are used to produce polymer-aluminum-silicon oxide composites. As an object for modeling, we refer to the mechanical properties of an aluminum-based alloy of the Al 2024 S type. Therefore, an anisotropic composite laminate modeled by a heterogeneous medium occupying a volume V composed of homogeneous phases $V^{(r)}$, $(r \in [0, N])$ with a characteristic volume function $\chi^{(r)}(\mathbf{x})$. In addition, inequality $V^{(r)} \square\ V$ holds, and the interphase contact is assumed to be ideal. The creep function of r th phase is denoted by $\mathbf{J}^{(r)}(t)$. Then the creep function of the composite can be represented as piecewise homogeneous:

$$\mathbf{J}(\mathbf{x}, \mathbf{t}) = \sum_{r=1}^{N} \mathbf{J}^{(r)}(t) \chi^{(r)}(\mathbf{x}). \tag{7}$$

Here and $\chi^{(r)}(\mathbf{x}) = 0, \quad \mathbf{x} \notin V^{(r)}$ in another case. Volumetric averaging over V and $V^{(r)}$ is denoted hereafter by $\langle \cdot \rangle$ and $\langle \cdot \rangle^{(r)}$, respectively. Volumetric averaging of the characteristic function by r -phase gives the volume concentration value $c_r = \langle \chi \rangle^{(r)}$. Volumetric averaging of any function f over the representative volume V and over the r-phase, Stochastic equilibrium equations and boundary conditions of the first linear approximation written in the domain of the LC transformations (3) can be represented in the form [5]

$$
\begin{aligned}
&\mathbf{L}(\nabla)\mathbf{v}\left(\mathbf{x},s\right) = -\nabla\boldsymbol{\tau}_{(1)}\left(\mathbf{x},s\right),\\
&\mathbf{v}(\mathbf{x},s) = \mathbf{u}_{(1)}(\mathbf{x},s) - \overline{\mathbf{u}}(\mathbf{x},s),\\
&\boldsymbol{\tau}_{(1)}\left(\mathbf{x},s\right) = \mathbf{f}\left(\mathbf{x},s\right)\mathbf{e}_{(1)}\left(\mathbf{x},s\right), \mathbf{f}\left(\mathbf{x},s\right) = \mathbf{C}\left(\mathbf{x},s\right) - \mathbf{L}.
\end{aligned} \tag{8}
$$

The dash bar above indicates the results of statistical averaging in a sample with a random elasticity tensor $\mathbf{C}(\mathbf{x},s)$, $\mathbf{L}$ is the elastic modulus tensor of a homogeneous body of comparison [6].

Second order nonlinear solution.

Following the proposed algorithm, we find the second order nonlinear solution, where field variables are expressed in terms of macroscopic deformations of the representative volume of the composite material

$$\mathbf{e}_{(1)}^{(r)} = \mathbf{A}^{(r)}\overline{\mathbf{e}},\ \ r \in [1, n+1],\ n+1 = m. \tag{9}$$

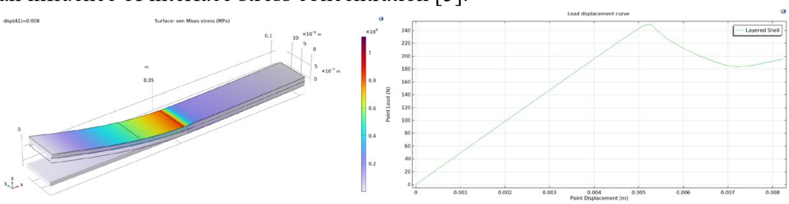
Superscripts in parentheses mean the result of the conditional statistical averaging operation. The first approximation of deformations is $\mathbf{e}_{(1)}^{(r)}$. By implementing the procedure defined by (), we obtain

$$\mathbf{e}_{(2)}^{(r)}(s) = \mathbf{A}^{(r)}(s)\overline{\mathbf{e}}(s) + \mathbf{A}^{(r)}(s)\left(\overline{\mathbf{e}}(s)\right),\ \ r \in [i \cup m],\ \ i \in [1,n], \mathbf{A}_m(s) = \left(\mathbf{I} + \mathbf{R}(s)\right)^{-1},$$

$$\mathbf{R} = \sum_{i=1}^{n} c_i \mathbf{R}_i,\ \ \mathbf{A}_i(s) = \left(\mathbf{I} + \mathbf{R}_i(s)\right)\mathbf{A}_m(s), \qquad \mathbf{R}_i = (\mathbf{zh})_{ii},\quad A_m(s) = -\mathbf{A}_m(s)\left(\sum_{i=1}^{n} c_i \mathbf{e}_\beta^i\right),$$

$$\mathbf{e}_\beta^i = (\mathbf{zh}_\beta)^i,\ \ \mathbf{h}_\beta^i = \boldsymbol{\beta}^i - \boldsymbol{\beta}^m, = -\sum_{i=1}^{n} c_i \mathbf{q}_i(s)\boldsymbol{\beta}_i(s); A_i(s) = \left(\mathbf{I} + \mathbf{R}_i(s)\right)A_m(s) + \mathbf{e}_\beta^i(s).$$

In the calculations with FEA [3, 4] it is assumed the all three phases are nonlinear elastic, an aluminum and epoxy are materials with hereditary creep properties. Several displacement and energy based failure criteria models are used, as well as an influence of interface stress concentration [5].



(a) (b)

Fig1. Results of FEA analysis of delamination in composite:
(a) Surface von Mises stress in lamina, (b) load – displacement curve.

As an example, consider a composite laminate, based on ED-6 resin, reinforced with alumina Al2024S and SiC fibers. Nonlinear elastic properties of constituents are presented in Table 1.

Table 1

Nonlinear elastic material constants, GPa, for the Al2024/SiC/Epoxy composite.

Material	E , GPa	ν	ν_1 , GPa	ν_2 , GPa	ν_3 , GPa
Al 2024 S	55.8	0.33	-115.0	-160.5	-108.8
SiC	440.3	0.171	-227.2	31.5	-170.75
Epoxy	3.15	0.382	13.3	4.09	-10.02

Conclusions:

The method of successive approximation is used to obtain the full system of the hereditary creep equations of the second order. The creep functions of laminate composite are found. Also, interface stress concentration parameters are determined. The given examples show the importance of the mutual influence of nonlinear elastic and creep properties of the composite components on long term fracture parameters. A practical result is a possibility of FEA modeling the long-term strength of composite multilayered structure.

References

1. Aboudi J., Arnold S., Bednarcyk B. Micromechanics of Composite Materials. – Elsevier. – Boston, 2013. – 1011 p.
2. Besson J., Cailletaud G., Chaboche J., Forest S. Non-Linear Mechanics of Materials. Springer. – New York, 2010. – 450 p.
3. Reddy, J, N. Mechanics of laminated composite plates and shells: theory and analysis. – CRC Press LLC. – Boca Raton, 2004. – 830 p.
4. Zienkiewicz O.C., Taylor R.L.The Finite Element Method for Solid and Structural Mechanics. – Elsevier. – Oxford, Sixth edition 2005. – 648 p.
5. Maslov B.P. Nonlinear Hereditary Creep of Isotropic Composites of Random Structure. Int. Appl. Mech. – 2022. – **58**. – N1, P. 75 –90.
6. Maslov B.P. Hereditary creep of isotropic composites of random structure under a complex stress state. Bulletin of Taras Shevchenko National University of Kyiv Series Physics and Mathematics. – 2021. – **3**. – P. 77 – 80.