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A.K. Ablesimov , professor
(National Aviation University, Ukraine)

Stability of automatic control systems of inertial objects

Considered the method of calculation and construction of the stability border and the region in the plane of two parameters.

Introduction. Performing by the automatic control systems (ACS) functions assigned to them is possible only when they are stable. If ACS is stable, all the transients caused by external influences will be damped and oscillations in a system will not arise. System stability is achieved only when there is a certain combination of its parameters, and provided with appropriate adjustments during maintenance.

Investigation of the influence of parameters on the stability of ACS management processes is performed during the system projecting. Solution of problem by constructing vectors of hodographs $\bar{A}(j\omega)$ or $\bar{W}_{\text{op}}(j\omega)$ for each change of a parameters is a very laborious work. It would be better to build the stability border in a space of variable parameters, which will divide regions of sustainable and unsustainable management. Further investigations will be reduced to the determination of the position of the operating point according to system parameters, which are interesting for the researcher, in one or another region.

Usually limited by two parameters of variation (A,B) for fixed values of the others. Stability region can be closed, that in general is not required. The equations of stability border may be defined with the help of the frequency criterias of Nyquist or Mikhailov

$$\left.\begin{array}{l} X(A,B,\omega)=0 \\ Y(A,B,\omega)=0 \end{array}\right\}; \quad \left.\begin{array}{l} U(A,B,\omega)=-1 \\ V(A,B,\omega)=0 \end{array}\right\}.$$

Solution of the problem. Analysis of automatic control systems of inertial control objects shows that different in design systems have congruent structure and transfer functions of the same type. This allows us to consider a methodology of analyzing their stability based on the generalized block diagram, shown in figure 1.

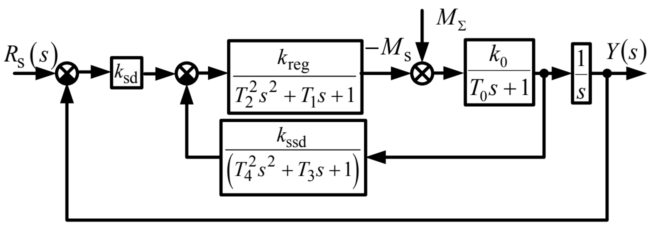

Fig.1 Generalized structural diagram of a typical automatic control system

In the block diagram the following notations are used:

$$W_0(s)=\frac{k_0}{(T_0 s+1)},\ W_{\text{reg}}(s)=\frac{k_{\text{reg}}}{T_2^2 s^2+Ts_1+1},\ W_{\text{ssd}}(s)=\frac{k_{\text{ssd}}}{T_4^2 s^2+T_3 s+1},\ W_{\text{sd}}(s)=k_{\text{sd}}$$

are the transfer functions of the inertial control object, regulator, sensor of speed deflection of control object, sensor of deflection of control object respectively; M_Σ is the total disturbing moment; M_{s} is the stabilization moment.

Transform a block diagram to the form shown in figure 2 selecting the channels of forming the stabilization moment. In the diagram introduced the notations of system stiffness G and damping D.

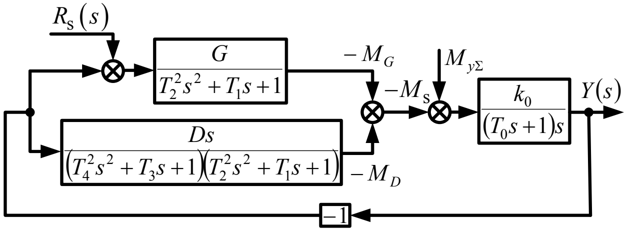

Fig.2 The structural diagram of a system with fission of forming channels of stabilization moment

Stiffness characterizes the ability of the system to counteract to the external disturbances. It is determined by the coefficients of strengthening of the deflection sensor and regulator $G=k_{\text{sd}}k_{\text{reg}}$.

Damping describes the ability of the system to put out vibrations arising in it. Damping depends from the coefficients of strengthening of the speed deflection sensor and regulator $D=k_{\text{ssd}}k_{\text{reg}}$.

Stiffness and damping directly influence on the formation of stabilizing moment and determines the effectiveness of systems countering to external disturbances. Will take stiffness G and damping D as variable parameters of system and find stability border and region in the plane of the selected parameters.

Let us find the transfer functions:

- by control signal

$$W_r(s)=\frac{k_0 G\left(T_4^2 s^2+T_3 s+1\right)}{A(s)},$$

- by external perturbation

$$W_m(s)=\frac{k_0\left(T_4^2 s^2+T_3 s+1\right)\left(T_2^2 s^2+Ts+1\right)}{A(s)}.$$

Here $A(s)$ is the characteristic polynomial of the sixth order

$$A(s)=a_6 s^6+a_5 s^5+a_4 s^4+a_3 s^3+\left(a_2+k_0 G T_4^2\right)s^2+k_0\left(D_\Sigma+GT_3\right)s+k_0 G.$$

Coefficients of the characteristic polynomial are

$$a_6 = T_4^2 T_2^2 T_0\ ;$$
$$a_5 = T_4^2 T_2^2 + T_4^2 T_1\ T_0\ + T_3\ T_2^2 T_0\ ;$$
$$a_4 = T_4^2 T_1\ + T_4^2 T_0\ + T_3 T_2^2 + T_3\ T_1\ T_0\ + T_2^2 T_0\ ;$$
$$a_3 = T_4^2 + T_3 T_1\ + T_3\ T_0\ + T_2^2 + T_1\ T_0\ ;$$
$$a_2 = T_3\ + T_1\ + T_0\ .$$

The total damping $D_\Sigma = D_0 + D$ is determined by the sum of constant damping $D_0 = 1/k_0 = f_0$, caused by natural friction of inertial control object, and of external damping D, which is generated by the sensor of speed deflection of control object.

Let us use the Mikhailov criterion for assessing the stability of the stabilization system

$$A(j\omega) = \left[-a_6\omega^6 + a_4\omega^4 - \left(a_2 + k_0 G T_4^2\right)\omega^2 + k_0 G\right] + j\left[a_5\omega^5 - a_3\omega^3 + k_0\left(D_\Sigma + G T_3\right)\omega\right].$$

Equating the real and imaginary parts to zero, we shall have

$$\left.\begin{aligned} X(\omega) &= -a_6\omega^6 + a_4\omega^4 - \left[a_2 + k_0 G_{\text{г}} T_4^2\right]\omega^2 + k_0 G_{\text{г}} = 0 \\ Y(\omega) &= a_5\omega^5 - a_3\omega^3 + k_0\left(D_\Sigma + G T_3\right)\omega = 0 \end{aligned}\right\},$$

where G_{sb} and $D_{\text{sb}\Sigma}$ are the stiffness and total damping, that corresponds the stability border $D_{\text{sb}\Sigma} = F\left(G_{\text{sb}}\right)$.

On the basis of last equations will just find dependences of the boundary values of stiffness and total damping from the frequency

$$G_{\text{sb}}(\omega) = \frac{a_6\omega^4 - a_4\omega^2 + a_2}{k_0\left(1 - T_4^2\omega^2\right)}\omega^2;$$

$$D_{\text{sb}\Sigma}(\omega) = \frac{a_3 - a_5\omega^2}{k_0}\omega^2 - G_{\text{sb}}(\omega) T_3.$$

According to the graph-analytical method [1] is constructed the stability border $D_{\text{sb}\Sigma} = F\left(G_{\text{sb}}\right)$ of the system (Figure 3).

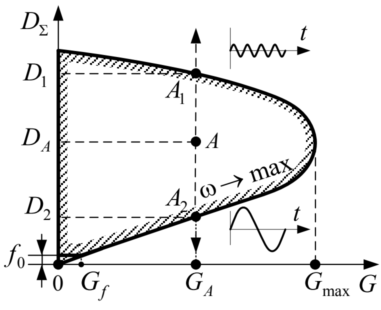

Fig.3 Region of stable management processes

Let us select the region of stability management, using the rule of applying shading [2]. For this we find the determinant

$$\begin{vmatrix} \dfrac{\partial X(G,D,\omega)}{\partial G} & \dfrac{\partial X(G,D,\omega)}{\partial D} \\ \dfrac{\partial Y(G,D,\omega)}{\partial G} & \dfrac{\partial Y(G,D,\omega)}{\partial D} \end{vmatrix} = \begin{vmatrix} k_0\left(1+T_4^2\omega^2\right) & 0 \\ k_0 T_3\omega & k_0\omega \end{vmatrix} = k_0^2\omega\left(1+T_4^2\omega^2\right) > 0.$$

Since the determinant is greater than zero, we put shading on the left side, moving in the direction of increasing frequency $\omega \to \max$.

Stability border identified in the plane of variable parameters stability region. Inside this region, any selected operating point A , with parameters G_A and D_A corresponds to stable (damped) management processes.

Conclusions. The resulting stability border and region allow us to make a number of important conclusions:

- with decreasing the damping stability ACS retained until you reach the lower border of the stability region, where the frequency is relatively small. On reaching the lower border low frequency oscillations with large amplitudes are arising in the system.

- with increasing the damping stability ACS retained until you reach the upper border of the stability region, where the frequency is relatively high. On reaching the upper border high frequency oscillations with small amplitudes are arising in the system.

- for a given value of stiffness, for example G_A, limits of changing damping are strictly limited $D_1 < D_A < D_2$.

- the border of the stability region defines the limit of stiffness $G_{\max}$, which can be obtained for a given system.

- if the system is missing the sensor of speed deflection of control object it will be stable only for a small value of stiffness G_f , that corresponds the damping by friction f_0 . Thus, the speed sensor expands the stability region of the system.

- the time constant $T_0 = J_0/f_0$ of the control object depends on its moment of inertia J_0 and is the scale of characteristics $G_{\text{sb}}(\omega)$ and $D_{\text{sb}\Sigma}(\omega)$. With the increasing of the moment of inertia is to the stability region expands proportionally. Therefore, to ensure stable mode of operation of ACS, where the control object has a large inertia, easier.

References

1. Ablesimov, O. K.; Alexandrov, E. E.; Alexandrova, I. E. 2008. Automatic control of moving objects and technological processes. Kharkov: NTU “KhPI”. 443 p. (in Ukrainian).
2. Ablesimov, O. K. 2014. Course of the theory of automatic control. Kyiv: Osvita Ukrainy. 270 p. (in Ukrainian).