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## Calculation of errors of movement of chains of measuring robots

This work is devoted to the calculation of errors in the movement of chains of the measuring robot by analyzing the possible trajectories of motion when working with small and large-sized parts.

## Analysis of the work of the measuring robot

In order to solve and organize the control of measuring robots, it is expedient that the grouping takes place on the basis of the same principles that can be used in solving the tasks of managing the robot and processing information. In robotics for these purposes, the method of homogeneous coordinates of points of three-dimensional space, which is based on the use of $4 \times 4$ dimension matrices, provides the definition of homogeneous coordinates of a point, a line, a plane and their description in a three-dimensional space.

In the tasks of estimating the probability of rotation of motion, the method of the interrelated coordinate image is widely used, which is as follows. Let $v+1$ and $v-$ be two coordinate systems (or two links of the manipulation system associated with these coordinate systems). The task of combining coordinate systems (or their associated units), which can be interpreted as the transformation of the coordinate system v to the coordinate system $v+1$ arises. The latter can be achieved by rotating, by two transitions, and again by rotating the coordinate system $v+1$ in the following sequence: rotation relative to the $z_{v}$ axis at an angle $\boldsymbol{\theta}_{z(v+1)}$ to ensure the parallelism of the $x_{v}$ and $x_{v+1}$ axes; moving along the $z_{v}$ by size $t_{z(v+1)}$ to align the coordinate of the system $v+1$ with the point of intersection of the axis $z_{v}$ a common perpendicular held up to the $z_{v}$ and $z_{v+1}$ axis; moving along the axis $x_{v+1}$ by size $t_{z(v+1)}$ to ensure the combination of the origin of coordinate systems; rotation relative to the axis $x_{v+1}$ on the corner $\boldsymbol{\theta}_{x(v+1)}$ to combine all the axes [1].

The homogeneous coordinates of a point $S$ in a three-dimensional space are any four numbers $x_{1}, x_{2}, x_{3}, x_{4}$, which do not all simultaneously equal zero and are related to its Cartesian coordinates $x, y, z$ by the equations $x=x_{1} / x_{2}, y=x_{2} / x_{4}$, $z=x_{3} / x_{4}$ which characterize homogeneity only at $\mathrm{x}_{4} \neq 0$. If $\mathrm{x}_{4}=0$, then the space point corresponding to the four numbers $x_{1}, x_{2}, x_{3}$ and 0 , may be infinitely distant in the direction of the vector $\vec{\gamma}=\left(b x_{1}, b x_{2}, b x_{3}\right), b \neq 0$.

Then the points with coordinates $(1,0,0,0),(0,1,0,0),(0,0,1,0)$ are infinitely distant from the corresponding axes $x, y, z$, and a point with coordinates $(0,0,0,1)$ corresponds to the origin of the coordinate system.

The equation $d x_{1}+e x_{2}+f x_{3}+g x_{4}=0$ describes a plane in a space that is infinitely distant with $d=e=f=0$.

Two independent equations

$$
\begin{gather*}
d x_{1}+e x_{2}+f x_{3}+g x_{4}=0 ; \\
d_{1} x_{1}+e_{1} x_{2}+f_{1} x_{3}+g_{1} x_{4}=0 \tag{1}
\end{gather*}
$$

define a line that is infinitely distant $d / d_{1}=e / e_{1}=f / f_{1}$. Operations over vectors in homogeneous coordinates are performed according to the same formulas as in the Cartesian, namely: the length of the vector $|\vec{r}|=\sqrt{d^{2}+e^{2}+f^{2}} /|\xi|$; adding and subtracting vectors $\vec{r}_{1}=[d, e, f, g]$ and $\vec{r}_{2}=\left[d_{1}, e_{1}, f_{1}, g_{1}\right]$ leads to the resulting vector

$$
\begin{equation*}
\vec{r}_{\text {рез }}=\left[\left(\frac{d}{g} \pm \frac{d_{1}}{g_{1}}\right),\left(\frac{e}{g} \pm \frac{e_{1}}{g_{1}}\right),\left(\frac{f}{g} \pm \frac{f_{1}}{g_{1}}\right), 1\right] \tag{2}
\end{equation*}
$$

the multiplication of the vector $\vec{r}_{1}=[d, e, f, g]$ on the scalar $p$ gives $\vec{r} p=[d, e, f, g / \vec{r}]$;
scalar and vector products of two vectors $\vec{r}_{1}$ and $\vec{r}_{2}$ have corresponding views

$$
\begin{align*}
& r_{c H}=r_{1} r_{2}=\left(d d_{1}+e e_{1}+f f_{1}\right) /\left(g g_{1}\right) \\
& r_{3}=\vec{r}_{1} \vec{r}_{2}\left[\left(e f_{1}-f e_{1}\right),\left(f d_{1}-d f_{1}\right),\left(d e_{1}-e d_{1}\right), g g_{1}\right] \tag{3}
\end{align*}
$$

The variable for steam with rotational motion is the angle $\theta$, and for pairs with translational displacement, the relative displacement $t$ [2].

Each of these movements corresponds to its transformation matrix, namely:

$$
\begin{align*}
& A_{v_{1},+1}^{y_{z}}=\left|\begin{array}{cccc}
\cos \theta_{z(v+1)} & -\sin \theta_{z(v+1)} & 0 & 0 \\
\sin \theta_{z(v+1)} & \cos \theta_{z(v+1)} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right| ; \quad A_{v_{1}, v+1}^{t_{z}}=\left|\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & t_{z(v+1)} \\
0 & 0 & 0 & 1
\end{array}\right| ;  \tag{4}\\
& A_{v_{1} v+1}^{t_{x}}=\left|\begin{array}{cccc}
1 & 0 & 0 & t_{x(v+1)} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right| ; \quad A_{v_{1} v+1}^{y_{x}}=\left|\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \cos \theta_{x(v+1)} & -\sin \theta_{x(v+1)} & 0 \\
0 & \sin \theta_{x(v+1)} & \cos \theta_{z(v+1)} & 0 \\
0 & 0 & 0 & 0
\end{array}\right| \text {. }
\end{align*}
$$

By multiplying the matrices corresponding to elementary motions when combining coordinate systems, we obtain the final matrix, which these systems are related to:

$$
\begin{gathered}
A_{v}=A_{v_{1} v+1}^{y_{z}} A_{v_{1} v+1}^{t_{z}} A_{v_{1} v+1}^{t_{x}} A_{v_{1} v+1}^{y_{x}}= \\
=\left|\begin{array}{cccc}
\cos \theta_{z(v+1)} & -\sin \theta_{z(v+1)} \cos \theta_{x(v+1)} & \sin \theta_{z(v+1)} \cos \theta_{x(v+1)} & t_{x(v+1)} \cos \theta_{z(v+1)} \\
\sin \theta_{z(v+1)} & \cos \theta_{z(v+1)} \cos \theta_{x(v+1)} & -\cos \theta_{z(v+1)} \sin \theta_{x(v+1)} & t_{x(v+1)} \sin \theta_{z(v+1)} \\
0 & \sin \theta_{x(v+1)} & \cos \theta_{z(v+1)} & t_{z(v+1)} \\
0 & 0 & 0 & 1
\end{array}\right| \text { (5) }
\end{gathered}
$$

We pass to the initial (absolute) $O_{x y z}$ and the final $O_{x^{\prime} z^{\prime} z^{\prime}}$ coordinate system, the position and orientation of the working organ of the measuring arm relative to the initial (base) of the coordinate system are written as:

$$
\begin{equation*}
T_{x^{\prime} y^{\prime} z^{\prime}}=T_{n} \prod_{v=1}^{N} A_{v} \tag{6}
\end{equation*}
$$

where $T_{n}$ - is a unit matrix.
If, however, the matrix $T_{n}$ characterizes the position of the coordinate system of the moving robot in some other coordinate system (for example, the technological equipment being serviced), then in this case it has the form of the matrix of the $T$ transform.

If Cartesian coordinates are given in one, for example, a moving coordinate system $O x^{\prime} y^{\prime} z^{\prime}$, then you can go to the description of the position of this point in another (for example, the base) coordinate system $x, y, z$.

The matrix of homogeneous transformation, which is connected by homogeneous points in moving and immobile coordinate systems, is a table of coefficients of the system of equations describing the position of a point in homogeneous coordinates [3].

The geometric content of the matrix of the robot is determined by its structure, in which the first three columns characterize the directions of the coordinate axes $x^{\prime}, y^{\prime}, z^{\prime}$, and the fourth is the vector $\vec{r}$ of the position of the origin of the coordinate system $O_{x^{\prime} y^{\prime} z^{\prime}}$ in the base coordinate system of the $O_{x y z}$ robot. In addition, the second and third columns of the matrix represent their own orientation vectors and approach to the object of the determination of coordinates.

## References

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