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### **Analysis of long term strength of gas turbine engine elements**

*The method of cyclic creep-damage problem's solution for aviation structural elements is presented. Finite element method and developed constitutive equations are used for the analysis of non-linear behavior at periodic modes of loading. The materials with and without creep recovery are considered. The results of numerical estimations of long term strength of gas turbine engine bodies are discussed.*

#### **Introduction**

For the structural elements of modern aircraft structural elements operating at elevated temperatures and in conditions of joint action of both static and cyclic loads, the processes of the development of irreversible creep strains and hidden damage accumulation are wide spread. Many of these elements correspond to the calculation schemes of thin shells of revolution. Due to the wide spread of such elements in modern aerospace engineering the questions of modeling the stress-strain state under cyclic loading at high temperatures are of great importance [1]. Modern aviation gas turbine engines (GTE) are designed for operating conditions characterized by a joint action of both long static and cyclic stresses that change with periods of high and relatively short duration (forced oscillations). In flight and operating cycles pressures and temperatures slowly increase from the start to the set mode, and when switched off, they decrease to the initial. At working temperatures of the GTE in the material of their structural elements there is an increase in the creep strains and accumulation of damage due to high temperature creep, low and high-cycle fatigue [2]. Cyclic stresses, even at low amplitudes, significantly affect the rate of growth of creep strains and the long-term strength of materials.

Most of the numerical studies of creep and damage of shell structures are carried out only for the case of static loads. A description of the method, algorithms and software for the numerical estimation of the cyclic load influence on creep-damage processes in plates and shells contains in [3, 4]. In real conditions of operation of structural elements their loading is a complicated process which is a combination of cycles of temperature and load, significantly differ in periods. In this regard, the development of a method for solving cyclic creep-damage problems can be regarded as relevant.

#### **Problem statement and method of solution**

The mathematical formulation of the problem of creep and damage of thin shells is given in [5]. To solve the problem, we choose the finite element method (FEM) with use of numerical time integration schemes.

Finite element formulation of the problem of creep and creep damage accumulation is determined by the following system of equations:

$$[K]\{u\} = \{F\} + \{F^c\} + \{F^n\}; \quad (1)$$

$$\begin{aligned} \{F\} &= \sum_{N_\beta} \int_S [N]^T \{p\} dS; & \{F^c\} &= \sum_{N_\beta} \int_{V_\beta} [B]^T \cdot [C] \cdot \{c\} dV; \\ \{F^n\} &= \sum_{N_\beta} \left( \int_S [N]^T \{p^n\} dS + \int_{V_\beta} [B]^T [C] \{\varepsilon^n\} dV \right); \end{aligned} \quad (3)$$

where  $K$  is stiffness matrix;  $u$  is the global nodal displacement vector;  $F$  is the vector of nodal external forces;  $F^c$  is the vector of nodal forces, induced by creep strains;  $C$  is matrix of elastic constants;  $N$  is the matrix of shape functions;  $p$  and  $P$  are surface and volume loads respectively;  $c$  is the vector of creep strains;  $\beta$  is the number of finite element as well as  $V_\beta$  is its volume;  $\sum_{N_\beta}$  is the sum on the total amount of finite elements;  $S$  is the area of the surface of loaded finite element;  $F^n$  is the vector of ‘fictitious’ forces from non-linear part of strains;  $p^n$  is the vector of nodal forces which determines by nonlinear part of elastic strains  $\varepsilon^n$ .

The system of equations (1) - (3) must be supplemented by constitutive equations describing the processes of creep and accumulation of creep damage under the cyclic loading.

The analysis of inelastic behavior is carried out under periodic load conditions for materials whose long term deformation is characterized by creep with or without recovery. The classical Bailey-Norton creep equations with Rabotnov-Kachanov damage evolution equation are used for standard materials. Creep with recovery is described by constitutive equations of Frederick and Armstrong.

In order to take into account the cyclical nature of the load by use the method of many scales with subsequent averaging over the period, the authors have developed new constitutive equations that identically describe the behavior of the material both for fast and slowly varying the cyclic components of the load [5, 6].

The cyclic creep-damage constitutive equations for material without essential recovery are the following [5]:

$$\begin{aligned} \dot{c}_{ij} &= B g_n K_n \frac{3(\sigma_i^0)^{n-1}}{2(1-\omega)^k} S_{ij}^0, \quad \dot{\omega} = D g_r K_r \frac{(\sigma_e^{\omega 0})^r}{(1-\omega)^l}, \quad \omega(0) = 0, \quad \omega(t_*) = 1, \\ g_n &= \int_0^1 \left( 1 + \sum_{k=1}^{\infty} M_k \sin(2\pi k \xi + \beta_k) \right)^n d\xi, \quad g_r = \int_0^1 \left( 1 + \sum_{k=1}^{\infty} M_k \sin(2\pi k \xi + \beta_k) \right)^r d\xi, \\ K_n &= \int_0^1 (1 + A_n \sin(2\pi \xi))^n d\xi, \quad K_r = \int_0^1 (1 + A_r \sin(2\pi \xi))^r d\xi, \quad A_n = \frac{A}{g_n^{1/n}}, \quad A_r = \frac{A}{g_r^{1/r}}, \end{aligned}$$

where  $A = \frac{\sigma_e^a}{\sigma_e^0}$ ,  $M_k = \frac{\sigma_e^{ak}}{\sigma_e^0}$  are the stress cycle asymmetry coefficients;  $S_{ij}^0$  are deviatoric components of stress tensor of basic motion  $\sigma_{ij}^0$ . For determining these

coefficients  $A$  and  $M_k$ , which are used in influence functions  $g_n$  and  $K_n$ , the static  $\sigma_e^0$  and amplitude equivalent stresses  $\sigma_e^a$ ,  $\sigma_e^{ak}$  are calculated.

Cyclic creep equations in which the creep recovery is considered, were obtained in [6] in the following form:

$$\dot{c}_{ij} = B \left[ F_1(\sigma_e^a) \sinh \left( D \left( \frac{3 \sigma_i^0 - \beta_i}{2 \bar{\sigma}_i} \right) \tilde{S}_{ij} \right) + F_2(\sigma_e^a) \cosh \left( D \left( \frac{3 \sigma_i^0 - \beta_i}{2 \bar{\sigma}_i} \right) \tilde{S}_{ij} \right) \right],$$

$$\dot{\beta}_{ij} = \frac{2}{3} C \dot{c}_{ij} \left( \dot{c}_{ij} - \dot{c}_i \frac{\beta_{ij}}{H_* F(M) \sigma_i^0} \right), \quad (5)$$

where  $\beta_{ij}$  is backstress and  $\tilde{\beta}_{ij}$  are its deviatoric components;  $\tilde{S}_{ij} = S_{ij} - \beta_{ij}$ ;

$$\bar{\sigma}_i = \sqrt{\frac{3}{2} \text{tr}(\tilde{S}_{ij}^2)}; \quad \dot{c}_i = \sqrt{\frac{2}{3} \text{tr}(\dot{c}_{ij}^2)};$$

$$F_1(\sigma_e^a) = \int_0^1 \cosh \left( D \sigma_e^a \sum_{k=0}^{\infty} (a_k \cos(2\pi k \xi) + b_k \sin(2\pi k \xi)) \right) d\xi,$$

$$F_2(\sigma_e^a) = \int_0^1 \sinh \left( D \sigma_e^a \sum_{k=0}^{\infty} (a_k \cos(2\pi k \xi) + b_k \sin(2\pi k \xi)) \right) d\xi$$

$$F(M) = \int_0^1 \left( 1 + M \sum_{k=0}^{\infty} (a_k \cos(2\pi k \xi) + b_k \sin(2\pi k \xi)) \right) d\xi, \quad M = \frac{\sigma_e^a}{\sigma_e^0}.$$

The presented averaged equations (4), (5) describe the processes of creep strain varying in a "slow" time. The impact of cyclic repetitive load is taken into account in the developed equations with the help of the corresponding influence functions.

### Numerical estimation of long term strength of aviation GTE bodies

The results of the numerical studies of cyclic creep of the gas turbine engine flue tube have made it possible to reveal the basic laws of its deformation and damage accumulation at a complex stress state.

With the help of the developed software complex [7], the considered tube is simulated by a combination of cylindrical and conical shells. Its finite element mesh is presented on Fig. 1.

The distribution of pressure in the combustion chambers of modern aircraft qualitatively corresponds to the form of the cycle, which is shown in Fig. 2. When the engine is running, vibrations of the tube walls may also occur due to the dynamic effects of other components and due to the effect of the gas flux acting on the walls.

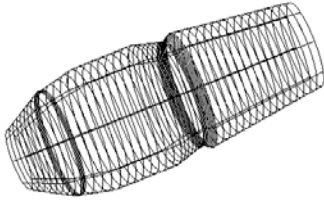


Fig. 1. Finite element model of gas turbine engine flue tube

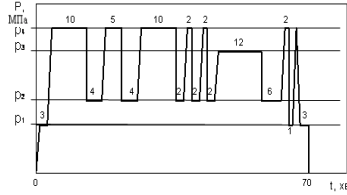


Fig. 2. Typical flight cycle

Thus, for the calculation of the stress-strain state and life time of the GTE gas tube under creep conditions, the combined effect of the static load, the load varying in the form of the cycle shown in Fig. 1 and the load from the vibration of the walls in the primary combustion zone is considered.

Calculations have been made for long-term strength estimation and time before the finishing of the hidden damage accumulation of the tube, which is equal to 660 hours, has been found.

Figures 3, 4 show the distribution of the damage parameter on the outer surface of the engine's flue tube for the time moment 10 hours and at the time of fracture, respectively.

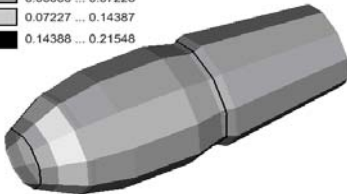
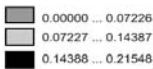


Fig. 3. Damage parameter distribution on tube outer surface,  $t=10$  h

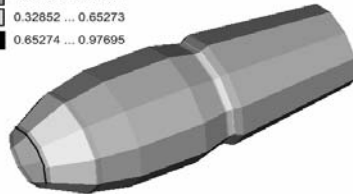
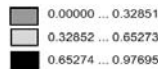


Fig. 4. Damage parameter distribution on tube outer surface,  $t=660$  h

Thus, as a result of numerical modeling of the cyclic creep problem, the parameters of the long term strength of the GTE flue tube and the place where the fracture occurs (combustion zone, Fig 4) were found. Analysis of the distribution of the damage parameter shows that in the shell structure there is another zone with its rather large values - the transition area between the primary and secondary zones ( $\omega = 0.58 - 0.65$ ). When some of the structural parameters and load values will be changed, it is very probable that the macrocracking will occur in this place.

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