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On non-Abelian groups with complemented non-Abelian subgroups

We present the description of locally finite non-Abelian groups in which all non-Abelian subgroups are complemented.

1. Introduction.

A subgroup A of the group G is called complemented in G if G contains a subgroup B such that $G=AB$ and $A\cap B=1$. As early as in 1937, Hall [1] studied finite groups with complemented subgroups. A complete description of arbitrary groups (both finite and infinite) with this property, called completely factorizable, was obtained later in 1953 by Baeva [2] (see also [3,4]). In the works by Chernikov [5] and Gorchakov [6] it was shown that arbitrary completely factorizable groups coincide with groups in which all Abelian subgroups are complemented. This, it turned out that the restriction of the system of complemented subgroups from all subgroups of the group to a system of abelian subgroups does not lead to an extension of the class of completely factorable groups. The question naturally arose of studying non-Abelian groups with complemented non-Abelian subgroups posed by Chernikov in [7].

In our works [8 - 10] locally finite, in particular, finite groups with complemented non-Abelian subgroups were studied. Thus, it turned out that they are solvable and their degree of solvability does not exceed 3. In [11], non-Abelian locally finite groups containing at least one non-Abelian Sylow subgroup in which all non-Abelian subgroups are complemented were studied. From its results follow two theorems 1 and 2 on the structure of nilpotent and non-nilpotent locally finite groups of this kind, respectively. And, finally, from the results of [12] imply Theorem 3 on the structure of locally finite non-nilpotent A-groups with complemented non-Abelian subgroups.

Following Hall and Taunt, locally finitely solvable groups with abelian Sylow subgroups are called A-groups (as in the finite case).

2. Lemmas.

Let G be an arbitrary non-Abelian group possessing the property: any non-Abelian subgroup of is complemented by. Then all non-Abelian subgroups and non-Abelian factor groups of the group G possess the same property. In addition, the factor group of the group G with respect to its non-Abelian normal divisor is completely factorizable.

Lemma 1. In an A-group the intersection of the center with the commutant is trivial.

Lemma 2. In a finite A-group the commutants of normal subgroups are complemented.

Lemma 3. If a commutant in a non-Abelian infinite binary finite group G with complemented nonabelian subgroups is finite, then $G = H \times B$, where H is a finite group with complemented non-Abelian subgroups, and B is an infinite completely factorizable abelian group.

Lemma 4. A locally finite non-Abelian group G with complemented non-Abelian subgroups is at most three-step solvable. In particular, if G is nilpotent, then $G''=1$.

Lemma 5. A non-Abelian nilpotent group with complemented non-Abelian subgroups is locally finite.

Lemma 6. A locally finite no nilpotent directly indecomposable not completely factorizable group G with infinite commutant and complemented non-Abelian subgroups contains an infinite maximum Abelian normal subgroup of finite index.

3. Nilpotent groups.

Theorem 1. If all non-Abelian subgroups are complemented in a non-Abelian nilpotent group G, then $G = H \times B$ and B is abelian completely factorizable, and H is a nonabelian primary (with respect to p) group of one of the 15 finite types of groups (their the description is given in [11].

4. Nonnilpotent groups.

Theorem 2. If in a locally finite non-nilpotent group G containing at least one non-Abelian Sylow subgroup all non-Abelian subgroups are complemented, then $G = H \times B$, where B is a completely factorizable Abelian group and H is a non-Abelian group of one of the 3 types of groups (their the description is given in [11].

Theorem 3. If in a locally finite non-nilpotent A-group G all non-Abelian subgroups are supplemented, then $G = H \times B$, where B – is a completely factorizable Abelian group and H is a non-Abelian group of one of the 3 types of groups (their the description is given in [12].

Question. The structure of binary finite groups with complemented non-Abelian subgroups.

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