UDC 519.872

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Algorithm of the Statistical Modeling of Retrial Queuing System GI / G / m / 0 / / K / D

A multichannel Lakatos-type retrial queing system with multi-capacity orbit and losses, i.e. a queuing system, in which a request gets (or doesn't get) service either at the time of arrival or after some cycle time T, has been studied. The algorithm of statistical modeling of the GI / G / m / 0 / / K / D queuing system has been developed.
Keywords — queing system; Lakatos-type queuing system; retrial queue; multichannel queuing system; cycling queuing system.

Introduction.

Classical *Queuing Theory* considers systems without blocking of requests: if a channel is free an arriving request is immediately being served. Obviously such systems represent an idealized picture of real processes. In many real systems an arriving request is being blocked until service conditions are being created, even if the channel is free at the moment of arrival. That's why the *Retrial Queuing Theory* [1] has been developed, thus allowing to apply new mathematical methods for the research of the real systems [2-6]. Ignoring the retrial effect can cause significant accuracy errors in research [7-8].

One of the important blocked-type systems are retrial queuing systems. An example of such a system is an airplane arrival and landing procedure. If the runway is busy upon airplane's arrival, the airplane is being dispatched to the hold. In aviation, holding (or flying a hold) is a maneuver designed to delay an aircraft already in flight while keeping it within a specified airspace [9]. The holding is delaying airplanes that have arrived but cannot land yet because of traffic congestion, poor weather, or runway unavailability. Several aircraft may fly the same holding pattern at the same time, separated vertically by 1,000 feet or more. This is generally described as a stack or holding stack. As a rule, new arrivals will be added at the top. The aircraft at the bottom of the stack will be taken out and allowed to make an approach first, after which all aircraft in the stack move down one level, and so on [10].

Obviously, holding will cause delays of aircrafts, i.e. creating queues. Thereby, the aircraft landing process can be described as a retrial queuing system with FCFS discipline [11] (or Lakatos-type retrial queuing system [12]): if there is any request in the orbit, it will be served earlier than the arriving new ones. The retrials from the orbit are served after some time, multiple to a given cycle time T.

In [13] authors have found an analytical result for the number of calls in a single-channel cycling-waiting system with exponential distributions of arrival and service times.

Problem statement.

Consider an m-channel Lakatos-type queuing system with a recurrent input flow of requests and an orbit of a K-place capacity and losses. The distribution functions of the intervals between arrivals and service times have general form. Let us denote these distribution functions as $A(x)$, $B(x)$ (hereafter x > 0); let $\int_0^{\infty} x\,dA(x) < \infty$, $\int_0^{\infty} x\,dB(x) < \infty$. Let waiting times in the orbit (orbit's cycle) have determined form $D(x) = T$. Therefore, by extended Kendall's notation [3], the GI / G / m / 0 / / K / D (Lakatos-type) queuing system has been considered.

Statistical modeling algorithm.

Let's describe the developed algorithm of the statistical modeling of the *GI* / *G* / *m* / 0 / / *K* / *D* (Lakatos-type) queuing system in order to obtain stationary loss probability.

Let's denote $t = 0$ — a moment of the arrival of the first request, t — current moment of time. Let's introduce auxiliary variables: $\lambda(t)$ — time from the moment of t to the next request from the initial flow, $v(t), 0 \le v(t) \le m$ — number of busy servers at the moment of t; $\mu_j(t), j = \overline{1,m}$ — times from the moment of t till the service completion of the requsts that are being served at $s \le v(t)$ service channels, and non-zero elements of $\{\mu_j(t), j = \overline{1,m}\}$ are sorted in ascending order, and zero-elements are placed at the end; $\gamma(t), 0 \le \gamma(t) \le K$ — number of requests in the orbit at the moment of t; $\nu_i(t), i = \overline{1,K}$ — times from the moment of t to the next retrial attempt of the request from the $l \le K$ orbit places to get served, and non-zero elements of $\{\nu_i(t), j = \overline{1,K}\}$ are sorted in ascending order, and zero-elements are placed at the end.

The initial conditions are: $\lambda(0) = 0$, $v(0) = 0$, $\gamma(0) = 0$, $\forall \mu_j(0) = 0$, $\forall \nu_j(0) = 0$.

Let t_k be the nodal modeling moments such as $t_k : 0 = t_0 < t_1 < \ldots$, at which some variable vanishes, i.e. either $\lambda(t_k) = 0$, or $\mu_1(t_k) = 0$, or else $\nu_1(t) = 0$, that means, correspondingly: (1) either a new request enters the system (a request from the initial flow), (2) or the service of a request has been completed (only if $v(t_k) > 0$), or else (3) a request from the orbit retries to get service (a request from the retrial flow; only if $\gamma(t_k) > 0$).

Let's suppose that all variables are continuous on the left.

Let's denote: $\lambda(t_k) = \lambda_k$, $\lambda(t_k + o) = \lambda_k^+$; $v(t_k) = v_k$, $v(t_k + o) = v^+{}_k$; $\mu_j(t_k) = \mu_{jk}$, $\mu_j(t_k + o) = \mu_{jk}^+$, $j = \overline{1,m}$; $\gamma(t_k) = \gamma_k$, $\gamma(t_k + o) = \gamma_k^+$; $\nu_i(t_k) = \nu_{ik}$, $\nu_i(t_k + o) = w_{ik}^+$, $i = \overline{1,K}$.

Let $\overline{\xi_k} = (\lambda_k, v_k, \mu_{jk} \mid j = \overline{1,m}, \gamma_k, \nu_{ik} \mid i = \overline{1,K})$ and $\overline{\xi_k^+} = (\lambda^+{}_k, v^+{}_k, \mu^+{}_{jk} \mid j = \overline{1,m}, \gamma^+{}_k, \nu^+{}_{ik} \mid i = \overline{1,K})$.

We model the system's behaviour by the scheme:

$$\xi_0 \to \xi_0^+ \to \xi_1 \to \xi_1^+ \to \xi_2 \to \xi_2^+ \to \dots$$

Let's describe the transition $\overline{\xi_k} \to \overline{\xi^+{}_k}$, $k \geq 0$:

1. If $\lambda_k = 0$ (a new request arrives) and at the same time (the new random value λ_k^+ with the general distribution function $A(x)$ is generated in each of the substeps)

a) either $v_k = 0, \gamma_k = 0$ or $0 < v_k < m, m > 1$, $\gamma_k = 0$, then the request goes to the server. Update the number of served requests. Generate a random value y_k with general distribution function $B(x)$, and place it between times μ_{jk} so that the ascending order preserves;

b) either $v_k = 0, 0 \leq \gamma_k < K$, or $v_k = m, 0 \leq \gamma_k < K$, or else $0 < v_k < m, 0 < \gamma_k < K$, then the request goes to the orbit, and $\gamma_k^+ = \gamma_k + 1$, $\nu_{ik}^+ = T$, where i is the index of the first zero-element in $\{\nu_{ik}, i = \overline{1,K}\}$ after non-zero element;

c) $0 \leq v_k \leq m$, $\gamma_k = K$, then the request is lost.

2. If $\lambda_k > 0$ and at the same time

a) $\mu_{ik} = 0, v_k > 0$ (a service completion of the request), then $v^+{}_k = v_k - 1$, $\mu^+{}_{jk} = \mu_{j+1,k}, j = \overline{1, m-1}$, $\mu^+{}_{mk} = 0$; and if at the same time $\nu_{1k} = 0$ and $\gamma_k > 0$, then the requests in the orbit make another retrial cycle;

b) $\nu_{1k} = 0$, $\gamma_k > 0$, then retrial request makes attempt to get service.

Let's describe the transition $\overline{\xi^+{}_k} \to \overline{\xi_{k+1}}$, $k \geq 0$. Let $\tau_k = \min\{\lambda_k^+, \mu_k^+ \mid v_k^+ > 0, \nu_k^+ \mid \gamma_k^+ > 0\}$ then

$$\lambda_{k+1} = \lambda_k^+ - \tau_k,$$

$$\mu_{j,k+1} = \begin{cases} \mu_{jk}^+ - \tau_k, \mu_{jk}^+ > 0, \\ 0, \text{ otherwise.} \end{cases}$$

$$\nu_{j,k+1} = \begin{cases} \nu_{jk}^+ - \tau_k, \nu_{jk}^+ > 0, \\ 0, \text{ otherwise.} \end{cases}$$

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