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**Sensitivity analysis of a quadcopter robust control systems to the wind disturbance**

*The analysis of the sensitivity and the complementary sensitivity functions of the quadcopter robust control system is considered. This system allows you to resist disturbances, especially wind. The mathematical model of the quadrocopter and the $H_\infty$-based regulator algorithm are also provided.*

**Actuality.** One of the cheapest unmanned aerial vehicle (UAV), which is used in various tasks, is a quadcopter. Such tasks can be like delivering mail, parcel, photo and video capturing, energy audit tasks, etc.

Flight time, precision of UAV position depends on control system quality and how this system with controlled object will react to external disturbances, like wind. As battery, electronic and environment characteristics are not static, they can unpredictably change, quality control, and behavior of system can change with them. This can lead to not-optimal control of system (that may cause big battery consumption) or system can become unstable. That's why robust control system was used for controlling quadcopter object. But for optimal designing of control system it is needed to make sensitivity analyze of system.

**Problem Statement.** To solve problem, it is necessary to define math model of the controlled object. Equation system that describes angular motion of the quadcopter UAV:

$$\begin{cases} (I_x+\Delta I_x)\dot{\omega}_x = U_x - (R_x+\Delta R_x)\omega_x + M_x - M_{px} - \\ -((I_z+\Delta I_z)\omega_z + (I_{Mz}+\Delta I_{Mz})\omega_\Sigma)\omega_y + (I_y+\Delta I_y)\omega_y\omega_z \\ (I_y+\Delta I_y)\dot{\omega}_y = U_y - (R_y+\Delta R_y)\omega_y + M_y - M_{py} + \\ +((I_z+\Delta I_z)\omega_z + (I_{Mz}+\Delta I_{Mz})\omega_\Sigma)\omega_x - (I_x+\Delta I_x)\omega_x\omega_z \\ (I_z+\Delta I_z)\dot{\omega}_z = U_z - (R_z+\Delta R_z)\omega_z + M_z + \\ +(I_{Mz}+\Delta I_{Mz})\dot{\omega}_{M\Sigma} - (I_y+\Delta I_y)\omega_y\omega_x + (I_x+\Delta I_x)\omega_y\omega_x \end{cases}, \quad (1)$$

where the relationship between the angular velocities of UAV in the Earth coordinate system and angular velocities in the system, connected to UAV:

$$\begin{cases} \omega_x = -\dot{\Psi}\cos\Phi\sin\Theta + \dot{\Phi}\cos\Theta \\ \omega_y = \dot{\Psi}\sin\Phi + \dot{\Theta} \\ \omega_z = \dot{\Psi}\cos\Phi\cos\Theta + \dot{\Phi}\sin\Theta \end{cases},$$

$\Psi$ – yaw angle, $\Theta$ – pitch angle, $\Phi$ – roll angle, $R_x, R_y, R_z$ – coefficients of air resistance to angular motion, $M_x, M_y, M_z$ – disturbing moments, $M_{px}, M_{py}$ –

pendulum moments that equal: $M_{px} = m_p g l_p \sin\Phi$, $M_{py} = m_p g l_p \sin\Theta$, $U_x$, $U_y$ and $U_z$ are controlling moments that equal: $U_x = \frac{d}{2}\left(\left(T_3 + T_4\right) - \left(T_1 + T_2\right)\right)$, $U_y = \frac{d}{2}\left(\left(T_3 + T_4\right) - \left(T_1 + T_2\right)\right)$, $U_z = b\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right)$, $T_i$ – lift force, that acts perpendicular to the propeller rotation plane and is equal to: $T_i = K_T \rho A\left(\omega_i R\right)^2 \sim \left(k + \Delta k\right)\omega_i^2$, $K_T$ – dimensionless coefficient, ρ – air density; $R$ – propeller radius, $\omega_i$ – angular speed of i-th engine, $k$ – total dimensionless coefficient, that includes other coefitients, $\omega_\Sigma = \omega_1 + \omega_2 + \omega_3 + \omega_4$ – the sum of angular velocities of all engines, $A$ – the area covered by the propeller, $m_p$ – pendulum mass, $g$ – acceleration of gravity, $l_p$ – the length of the center of the mass of the pendulum. All uncertainty are shown as deviations with $\Delta$ symbol.

Linearized UAV in state space with unknown parameters deviations of plant:

$$\begin{cases} \frac{d}{dt}\vec{X}(t) = \left(A + \alpha(t)\right)\vec{X}(t) + \left(B + \beta(t)\right)\vec{U}(t) \\ y = C\vec{X}(t) \end{cases},$$

where

$$\vec{X}(t) = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \Phi \\ \Theta \\ \Psi \end{bmatrix}, \vec{U}(t) = \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}, A = \begin{bmatrix} -\frac{R_{mx}}{I_x} & 0 & 0 & -\frac{m_p g l_p}{I_x} & 0 & 0 \\ 0 & -\frac{R_{my}}{I_y} & 0 & 0 & -\frac{m_p g l_p}{I_y} & 0 \\ 0 & 0 & -\frac{R_{mz}}{I_z} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} -\frac{k}{I_x} & \frac{k}{I_x} & -\frac{k}{I_x} & \frac{k}{I_x} \\ \frac{k}{I_y} & \frac{k}{I_y} & -\frac{k}{I_y} & -\frac{k}{I_y} \\ \frac{b}{I_z} & -\frac{b}{I_z} & \frac{b}{I_z} & -\frac{b}{I_z} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where $x \in R^n, u \in R^r, r \le n$, The initial state belongs to an unknown subset, i.e. $x(t_0) \in X_0$, $\alpha(t)$ and $\beta(t) \in \Omega$ – the unknown real matrix functions on $t \in [t_0, T]$ with parameters of object uncertainty (deviations from the values of parameters of an ideal system [1,2].

Basic block diagram that was used in this work is shown on Fig.1. [3,5].

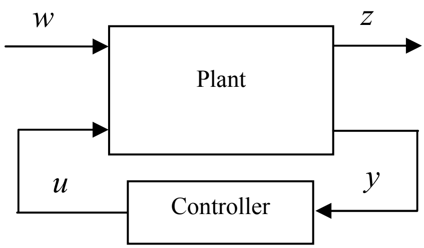


Fig. 1. Block diagram

On Fig.1. is shown *w* is input (setting impact of system, disturbance and noise), *z* is output, *u* is control impact and *y* is data from sensors. Sensitivity transfer function is transfer function from reference input to tracking error [3]. It shows output response behavior on input control signal.

Accordingly, to [4] there are four sensitivity functions in case of single feedback: $\frac{1}{1+PC}$ – sensitivity function, $\frac{P}{1+PC}$ – load (disturbance) sensitivity function, $\frac{PC}{1+PC}$ – complementary sensitivity function, $\frac{C}{1+PC}$ – noise sensitivity function, where C – is regulator, P is plant. Also if we add sensitivity function and complementary function we will receive next equation:

$$S + T = \frac{1}{1+PC} + \frac{PC}{1+PC} = I, \tag{2}$$

where $I$ is unit matrix, $S$ – sensitivity function, $T$ – complementary sensitivity function. It set restriction on system quality.

For retrieving optimal control signal tracking and good noise rejection, we need to keep S small. Also we need to keep T small for rejecting influence of object uncertainty, that influence on system, and sensor noises. As input control of quadcopter system is located in low frequency domain and noises are located in high frequency, we need to keep $S$ small in low frequency domain and $T$ keep small on high frequency domain [6].

Control system can be also represented by the following equation [5,7]:

$$P = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right], \tag{3}$$

where $P$ is the multidimensional transfer function, that optimize from $\begin{bmatrix} W & U \end{bmatrix}^T$ to $\begin{bmatrix} X & Y \end{bmatrix}^T$, $B_1$ - disturbance matrix, $B_2$ is input matrix, $C_1$ - output matrix of errors to be kept small, $C_2$ - output measurement matrix, W – disturbance vector.

For controlling quadcopter was designed [2] $H_\infty$-controller, that minimize $H_\infty$ norm $T_X^W$ from $W$ to $X$ [7]:

$$\left\| T_X^W(s) \right\|_{H_\infty}^{def} = \sup_{c>0} \sup_{\omega} \sqrt{\lambda_{\max}\left( T_X^W(c - j\omega) \right)^T T_X^W(c - j\omega)},$$

where $\lambda_{\max}$ is eigenvalue of $\left( T_X^W(c - j\omega) \right)^T T_X^W(c - j\omega)$.

Quality index of control of regulator, that will provide decreased influence of disturbances:

$$J(K) = \left\| T_X^W(s) \right\|_\infty ; \; J(K_{opt}) = \inf_K \left\| T_X^W(s) \right\|_\infty = \gamma_{opt} .$$

Suboptimal regulator, that was received when $\left\| T_X^W(s) \right\|_\infty < \gamma$:

$$K_\infty(s) = \begin{bmatrix} A + \gamma^{-2} B_1 B_1^T X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2 & -Z_\infty L_\infty \\ F_\infty & 0 \end{bmatrix}, \qquad (4)$$

**Conclusion.** In this work was done analyze of sensitivity functions of quadcopter control system and showed relation between control system and them. Designing of control system with considering of knowing sensitivity functions behavior can give desired optimal control.

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