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Modelling of creep and fatigue life of viscoelastic metal matrix composites

Correspondence principles for quasilinear viscoelastic material proposed to determine the relations between the current stress, or strain, and the instantaneous elastic stress, or strain. Stress concentration near inclusions evaluated, and fatigue criteria Hashin's type for composite material used for fatigue life prediction and nondestructive control problems of composites.

Metal matrix composites (MMC) used in aviation structures can experience fatigue damage and failure due to the repeated loads. Theoretical estimation of remaining lifetimes and residual strength is an important problem of solid mechanics and mathematic modeling. The response of composite structures under fatigue loading is a problem that has led to the development of a numbers of fatigue prediction models. The focus of this paper is on the strength degradation effects, continuum damage mechanics approach, and micromechanics model capabilities [1-3].

A commonly used approach in fatigue life predictions is to use stress versus life, known as $S-N$ curves. The constant amplitude cyclic loads are characterized by the mean stress level σ^m and the amplitude σ^a of the stress variations around the mean. This is alternatively expressed in terms of the maximum stress and the stress ratio or R-ratio. The situation is more complex in the case of heterogeneous media, strong stress triaxiality, and rheology time presence. For the analysis of creep fatigue problems in the framework of the quasi-linear viscoelasticity model, we use the correspondence principle, which is different from that used in the linear theory [4]. In this case, there is no assumption of an analogy between the defining relations of nonlinear elasticity and nonlinear viscoelasticity. Let t be the time, $\mathbf{x}$, $\mathbf{y}(\mathbf{x},t)$, $\mathbf{e}(\mathbf{x},t)$ and $\mathbf{u}(\mathbf{x},t)$ be the position, the current stress, the current strain, and the current displacement in three-dimensional case, respectively. We assume that the viscoelastic material possesses instantaneous elastic response $\mathbf{y}^{el}(\mathbf{x},t)$, $\mathbf{e}^{el}(\mathbf{x},t)$, $\mathbf{u}^{el}(\mathbf{x},t)$. The model requires that the loading curves and the unloading curves must fall in the same curve, and the stress and the strain must return to the origin simultaneously. It follows that there exists a strain energy function $W(\mathbf{e},\mathbf{x},t)$ with the property that

$$W = W(\mathbf{e}^{el},\mathbf{x},t), \quad \mathbf{y}^{el} = \frac{\partial W}{\partial \mathbf{e}^{el}}. \tag{1}$$

This equation defines the nonlinear elastic constitutive relations. To formulate the correspondence principles, we write down the constitutive equations of quasi-linear viscoelasticity between the current stress $\mathbf{y}(t)$ and $\mathbf{y}^{el}(t)$ the instantaneous (elastic) stress

$$\mathbf{y}^{el}(t) = \mathbf{g} \star d\mathbf{y} = \int_{-\infty}^{t} \mathbf{g}(t - t_1) d\mathbf{y}(t_1), \quad \mathbf{g} = \mathbf{J}(t) / \mathbf{J}(0) \;, \tag{2}$$

and constitutive relations for creep

$$\mathbf{e}^{el}(t) = \mathbf{ш}[\mathbf{y}(t)] = \mathbf{h} \star d\mathbf{e} = \mathbf{e} \star d\mathbf{h} = \mathbf{h} \star d\mathbf{e}, \quad \mathbf{h} = \mathbf{E}(t) / \mathbf{E}(0) \;. \tag{3}$$

Quasi-linear viscoelasticity allows generalizing the classical approaches in mechanics of composites [1]. We use here the enhanced viscoelastic model with internal parameter of stored damage D [3]. The local and overall constitutive relations between the infinitesimal strain $\mathbf{e}(\mathbf{x},t)$ and the Cauchy stress $\mathbf{y}(\mathbf{x},t)$ fields can be expressed as hereditary integrals. At the micro-scale of individual r constituents these are presented by [5]

$$\mathbf{e}(\mathbf{x},t) = (\mathbf{q}_r \star \mathbf{e}^{el})(\mathbf{x},t), \quad \mathbf{x} \in v_r \;. \tag{4}$$

Space coordinate $\mathbf{x}$ denotes a material point within any phase r of the composite and $\star$ stands for the Stieltjes convolution product. Similarly, the macroscopic or effective constitutive relations can be written as

$$\langle \mathbf{e} \rangle (t) = (\tilde{\mathbf{q}} \star \left\langle \frac{\partial U}{\partial \mathbf{y}} \right\rangle)(t) \tag{5}$$

Here $\langle \mathbf{e} \rangle (t)$ and $\langle \mathbf{y} \rangle (t)$ are the macroscopic, or averaged, strain and stress, the angle brackets denote spatial averaging over a representative volume element of the material. Four order tensors $\mathbf{q}_r(t)$ and $\tilde{\mathbf{q}}(t)$ are the local in phase r and effective creep reduced functions of the composite, respectively. In Fig. 1, it is shown how a visco-plastic model can be derived from a plastic model, by adding a dashpot. The resulting model is the so-called generalized Bingham's model. The original Bingham's model involves neither the spring assembled in series (E , no instantaneous elastic behavior, this is a rigid visco-plastic model) nor the spring in parallel ($H = 0$, no hardening). The elastic nonlinear strain is characterized by the spring $E(e)$, the visco-plastic strain, denoted by e^{vp} , is illustrated by the parallel assembly of the friction device and the dashpot. The equations of the model are obtained by combining all the elementary subsets $X = He^{vp}$, $\sigma^{v} = \mu^{r} \dot{e}^{vp}$, $\sigma^{p} < \sigma_{y}$ where X, σ^{v} and σ^{p} are respectively the stresses in the spring H , in the dashpot and in the friction device, and $\sigma = X + \sigma^{v} + \sigma^{p}$. An elastic domain is then present in this visco-plastic model. The border of the domain is reached when $\sigma^{p} = \sigma_{y}$.

The strain equivalence hypothesis, which states that any deformation behavior, whether uniaxial or multi-axial, of a damaged material is represented by the constitutive laws of the virgin material in which the usual stress $\sigma(t)$ is replaced by the so-called effective stress $\tilde{\sigma}(t)$, which enables the definition of an effective stress

$$\tilde{\sigma}(t) = \sigma(t)(1 - D)^{-1} \,. \tag{6}$$

In our model, the viscoelastic strain energy function $W(t)$ is coupled with damage parameter D. The expression of $W(t)$ is defined as [2]

$$2W(\mathbf{e},t)=(1-D(t))\int_{-\infty}^{t}\int_{-\infty}^{t}\frac{\partial \mathbf{e}(t_1)}{\partial t_1}\mathbf{E}(2t-t_1-t_2)\frac{\partial \mathbf{e}(t_2)}{\partial t_2}dt_1dt_2, \qquad (7)$$

where $\mathbf{E}(t)$ is relaxation tensor. The internal scalar variable D models the damage, which is assumed to be isotropic and varies between 0 for undamaged material and 1 under complete failure. The thermodynamic force associated with D is denoted $Y=-\partial W/\partial D$. The constitutive equation may be written in the compliance formulation to describe creep phenomena

$$2(1-D(t))U(\mathbf{s},t)=\int_{-\infty}^{t}\int_{-\infty}^{t}\frac{\partial \mathbf{y}(t_2)}{\partial t_2}\mathbf{J}(2t-t_1-t_2)\frac{\partial \mathbf{y}(t_1)}{\partial t_1}dt_1dt_2. \qquad (8)$$

According to (5) in quasi-linear viscoelasticity, for the proposed viscoelastic model coupled with damage the expression of stress is written as

$$\mathbf{y}(t)=(1-D(t))\int_{-\infty}^{t}\mathbf{h}(t-t_1)\frac{\partial W(\mathbf{e},t_1)}{\partial t_1}dt_1. \qquad (9)$$

The stress $\mathbf{y}(t)$ is thus related to the damage variable $D(t)$ and to the whole history of viscoelastic strains $\mathbf{e}(t)$ throw the energy $W(\mathbf{e},t)$ via Boltzmann's hereditary integral. Note that the constant volume concentration of phases remains unchanged after transforming from the time domain to the Carson domain. The Fortran95 programs from NAG-Fortran library we use for numerical analysis required. Statistical averaging of expressions is performed to define the mean deformation of short inclusions randomly oriented in volume. The result is that overall response of such a composite is isotropic [3]. Stress concentration near inclusions and overall creep response are modeled in the three-component metal matrix composite with aluminum viscoelastic matrix [1].

In this work, we use Hashin's [2] failure criteria to determine the fiber and matrix failures in a multicomponent composite. Equations that summarize the failure envelopes for fiber and matrix failure are obtained from Hashin's criteria. Short fibers and matrix failure in tension will be

$$\left(\frac{\sigma_{11}}{X_T}\right)^2+\frac{\sigma_{12}+\sigma_{13}}{S_{12}^2}=1\,,\;\frac{(\sigma_{22}+\sigma_{33})}{Y_T^2}+\frac{\sigma_{23}^2-\sigma_{22}\sigma_{33}}{S_{23}^2}+\frac{\sigma_{12}^2+\sigma_{13}^2}{S_{12}^2}=1\,. \qquad (10)$$

In equations (10), X_T and X_C are the longitudinal tensile and compressive strengths, Y_T and Y_C are the transverse tensile and compressive strengths, S_{12} is the in-plane shear strength, and S_{23} is the out of plane shear strength. An instantaneous matrix stiffness degradation scheme is used for the progressive failure when matrix or fiber failure is detected. We evaluate here the residual stiffness of the representative volume following failure in each mode [2]. In other words, the fatigue model used here is based on stiffness and strength reduction directly applied to the engineering stiffness constants and strengths that are RVE properties. To quantify and visualize the level of damage, a measure of the relative reduction in the stiffness/strength parameter due to damage D_P is calculated using equation (10)

$$D_P = 1 - \frac{P_r}{P_{init}} . \tag{11}$$

The non-linear cumulative damage rule for isotropic viscoelastic composite materials is used here. Scalar damage variable $D(t)$ evolves with the number of cycles. The evolution of damage is governed by increment methods [5]

$$\int_{D(k-1))}^{Dk} dD = \int_0^N [1-(1-D)^{1+\beta f}]^{\alpha f} (\frac{\sigma_k}{1-D})^{\beta f} \, dN \tag{12}$$

N is the number of cycles at the current stress state σ_k, D_k and D_{k-1} are the amount of damage after the current, and previous cycles, respectively, β_f is a material parameter, and α_f is a function of the current triaxial stress state [6].

The fatigue life of composites is evidently connected with stress concentration on the interphase surfaces. To present the formulation of the general interface model we introduce the following normal н and tangent з projection tensors of second order

$$\text{н} = \mathbf{n} \otimes \mathbf{n}; \quad \text{з} = \mathbf{1} - \text{н}. \tag{13}$$

Symbol $\mathbf{1}$ is the 3D second-order identity tensor. Let us construct further the normal $\mathbf{N}$ and tangent $\mathbf{T}$ projection tensors of fourth order by

$$\mathbf{N} = \mathbf{I} - \mathbf{T}; \quad \mathbf{T} = \text{з} \otimes \text{з} , \tag{14}$$

$\mathbf{I}$ is the fourth-order identity tensor for the space of second-order symmetric tensors. In fact, $\mathbf{T}$ and $\mathbf{N}$ correspond to the exterior and interior projection operators of Hill [3]. Next, we write

$$N_{ijkl} = \frac{1}{2}(\delta_{ik}\nu_{jl} + \delta_{jk}\nu_{il} + \delta_{il}\nu_{jk} + \delta_{jl}\nu_{ik}) - \nu_{ij}\nu_{kl},$$

$$T_{ijkl} = \frac{1}{2}(\eta_{ik}\eta_{jl} + \eta_{jk}\eta_{il})$$

$$\Gamma_{ijkl} = 2\mu^{-1}\left(N_{ijkl} - \frac{\nu}{1-\nu} n_i n_j n_k n_l\right); \quad \Pi_{ijkl} = 2\mu\left(T_{ijkl} + \frac{\nu}{1-\nu}\eta_{ij}\eta_{kl}\right),$$

$$\mathbf{\Gamma}(\mathbf{n}) = (\mathbf{N}\ \mathbf{E}\ \mathbf{N})^{-1}, \quad \mathbf{P}(\mathbf{n}) = (\mathbf{T}\ \mathbf{J}\ \mathbf{T})^{-1},$$

$$\mathbf{\Gamma}(\mathbf{n}) = \frac{1}{2}(\mathbf{G}(\mathbf{n}) \otimes \mathbf{N} + \mathbf{N} \otimes \mathbf{G}(\mathbf{n})) , \tag{15}$$

where the second-order tensor $\mathbf{G}$ is calculated by $\mathbf{G} = \mathbf{Q}^{-1}$, $\mathbf{Q} = \mathbf{nE\,n}$. In addition, the tensors $\mathbf{\Gamma}(\mathbf{n})$, $\mathbf{P}(\mathbf{n})$, relaxation function $\mathbf{E}(t)$ and creep function $\mathbf{J}(t)$ are connected by the identity $\mathbf{JP} + \mathbf{\Gamma E} = \mathbf{I}$.

Some numerical examples were analyzed. Properties of fibers and matrix are presented in Table 1. It should be noted that results of fatigue life prediction with the model proposed are in an acceptable correlation with known from literature experimental data.

Table 1.

Constituent nonlinear elastic material constants, GPa, for the B/SiC/Al2024 composite.

Material	E, GPa	ν	ν_1, GPa	ν_2, GPa	ν_3, GPa
Boron	467.3	0.361	-840.0	-420.0	-390.0
SiC	440.3	0.171	-227.2	31.5	-170.75
Al2024	80.34	0.296	-115.0	-160.5	-108.75

As a conclusion, we may notice that the viscoelastic model with internal parameter of stored damage suggested here be useful for long-term durability prediction and nondestructive control problems of composite elements.

References

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