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### **Analysis of destabilizing vibrational effects on a coordinate measuring machine**

*The report deals with theoretical studies of the influence of vibration disturbances on the part of the environment on coordinate measuring machines, their analysis has been carried out, which makes it possible to calculate the magnitude of the vibrational influences and to make corrections to the measurements promptly.*

The versatility and high precision during measurements made it possible to use coordinate measuring machines (CMM) in high-tech industries: aviation, shipbuilding, automobile, space and other industries. For efficient use, they are included in the composition of production lines and provide direct control of products.

In addition to the benefits mentioned above, stationary CMM require the creation of certain conditions that, in the production environment, are not always possible to provide measurements.

One of the components of the overall influence of the production environment on the measurement results is the vibration from the working equipment and heavy transport, which are transmitted through the floor at relatively long distances. They complicate the calibration of the CMM and make certain errors in the measurement. Therefore, an urgent question is the objective assessment of the magnitude of such vibrational effects, both in the calculation of means of protection against vibration of the environment, and in the measurement process: if the value of vibrations exceeds the design threshold, then it is necessary to stop measuring and take additional measures to compensate for the vibrational effects.

Consider the effect of vibrational vibrations of the floor on the contact area of the measuring tip on the example of the CMM of the portal layout (Fig. 1).

The CMM measuring block is a rigid portal structure that moves in parallel with the base and associated with aerostatic supports. The console with the measuring head is attached to the beam of the portal on horizontal, horizontal and vertical carriages.

Consider the static loads and moments of the specified nodes. The mass of the design of the mobile console with the mechanism of the measuring head will be called the combined mass. We will assume that the whole combined mass is centered on the beam. Deformation of the loaded beam at each of its points linearly depends on the distance to the fixing point  $k$  (Fig. 1). It is clear that for the selected design of the CMM the maximum deformation will be in the center of the beam ( $L/2 < k < L$ ).

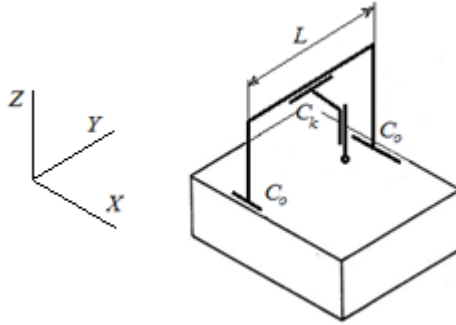


Fig.1. Kinematic scheme of coordinate measuring machine of portal layout.

$$\Delta_p = \frac{P(L - k)}{EJ} \quad (1)$$

$\Delta_p$  - deformation at the point with coordinate  $k$  relative to the base of the beam,

$P$  - weight of the console with the mechanism of the measuring head,

$L$  - length of beam,

$E$  - is the elastic modulus of the material from which the beam is made,

$J$  - is the geometric moment of inertia of the beam section relative to the axis, perpendicular to the gravity and the axis of the beam.

Substituting in (1) the numerical values of its constants, it is possible to calculate the elastic displacement of the middle of the beam  $s$ .

To do this, we integrate two expressions (1) to determine the deformation of the beam.

$$S_{max} = \frac{P}{EJ} \frac{(L)^3}{48}, \quad (2)$$

where  $S_{max}$  - deformation of the loaded beam.

Consider the deformation of the beam torsion. The deformation of the torsion of the loaded beam is proportional to the torsion moments arising from the force of gravity. This moment is created by a combined mass, since it is fixed asymmetrically in relation to the beam.

$$\Delta_k = \frac{P * L/2}{\frac{1}{3} * EJ}, \quad (3)$$

where  $\Delta_k$  - deformation of the torsion of the loaded beam.

The total elasticity of the base-portal-console design  $\Delta$  is equal to the weight ratio of the combined mass to the elastic displacement, which consists of bending

the beam and its torsion:

$$\Delta = \frac{P}{\Delta_p + \Delta_k} \quad (4)$$

Thus, if you make a static calculation on a particular model of the CMM, then it is possible to determine the deflection of the portal structure in the static under the action of gravity.

Consider the portal management of the CMM under the influence of external vibrational effects transmitted through the floor on.

With forced oscillations of the basis of the CMM the console with the mechanism of the measuring head makes fluctuations relative to it due to the elasticity of the portal structure (including aerostatic supports and carriages of vertical and horizontal drives).

Therefore, the task of estimating the value of the oscillation of the measuring head is reduced to the consideration of its forced oscillations in relation to the basis of the CMM in vibrations of the latter.

For each of the axes, in the three-dimensional coordinate system, the oscillations of the basis of the CMM are given by equations

$$\begin{cases} m(x_x + A \sin \omega t)'' = -c_x x_x \\ m(x_y + A \sin \omega t)'' = -c_y x_y \\ m(x_z + A \sin \omega t)'' = -c_z x_z \end{cases} \quad (5)$$

Where:

$x$  - deviation (displacement) of the united mass from the equilibrium position relative to the basis of the KVM,

$m$  - combined weight (console with carriages of horizontal and vertical displacements of a measuring head,

$C$  - stiffness of design,

$A$  - is the amplitude of periodic oscillations of the basis,

$\omega$  - circular frequency of oscillations of the base,

$t$  - is the time.

Let us rewrite the equation (5) in the form

$$\begin{cases} m\ddot{x}_x = -c_x x_x + Am\omega^2 \sin(\omega t) \\ m\ddot{x}_y = -c_y x_y + Am\omega^2 \sin(\omega t) \\ m\ddot{x}_z = -c_z x_z + Am\omega^2 \sin(\omega t) \end{cases}, \quad (6)$$

where  $\ddot{x}_x, \ddot{x}_y, \ddot{x}_z$  are the vibrational accelerations of the combined mass along the XYZ axes (the second derivative of the displacement  $x$ ).

Note that the stiffnesses  $c_x, c_y, c_z$ , with equal dimensions of the cross-section of the portal beams will be conditionally equal, and aerostatic supports have the same rigidity of the air gap on the axes of the XYZ.

Then:

$$\begin{aligned} c_x &= c_y = c_z, \\ c_k &= \sum c_x c_y c_z, \\ c &= c_k + 2c_0, \end{aligned}$$

where:

$s$  - the general firmness of the combined mass,

$c_k$  - total stiffness of all console combinations,

$c_0$  - rigidity of aerostatic supports.

Note that the stiffnesses  $c_x, c_y, c_z$ , with equal dimensions of the cross-section of the portal beams will be conditionally equal, and aerostatic supports have the same rigidity of the air gap on the axes of the XYZ.

We define the vibrational acceleration of the combined mass  $\ddot{x}$  as the sum of the vibrational accelerations along the axes of the XYZ:

$$\ddot{x} = \sqrt{\ddot{x}_x^2 + \ddot{x}_y^2 + \ddot{x}_z^2}.$$

We rewrite the equation (6) in a generalized form (true for all axes).

$$m\ddot{x} = -cx + Am\omega^2 \sin(\omega t) \quad (7)$$

where  $\ddot{x}$  is the vibrational acceleration of the combined mass (the second derivative from the displacement  $x$ ).

Simplify the expression (7) using the vibration acceleration formula for the basis of CMM:

$$a = -\omega^2 A \sin(\omega t) \quad (8)$$

It should be noted that the acceleration is shifted by 180 degrees in relation to the displacement  $x$ , as indicated by the negative sign of the sine.

We carry out the operation with the signs in the equation (8):

$$-a = \omega^2 A \sin(\omega t)$$

We substitute this value into equation (7) and define the displacement of the combined mass  $x$ :

$$x = \frac{-m(a + \ddot{x})}{c} \quad (9)$$

As can be seen from expression (9), the displacement  $x$  of the combined mass directly proportional to the sum of accelerations of the combined mass and the basis of the CMM.

Separately, such  $a$  parameter of vibrational influence as sharpness  $u$ , which characterizes the rate of acceleration change, should be considered.

Sharpness is the third derivative of moving  $x$ . This parameter allows you to consider the rate of change in the acceleration of the increase or decrease in the transfer of energy vibrational impact on the object.

$$u = -\omega^3 A \sin(\omega t) \quad (10)$$

From the expressions (8) and (11) we define the relationship between  $u$  and  $a$ :

$$u = a\omega$$

Then the expression (9) has the form:

$$m\ddot{x} = -cx - \frac{u}{\omega}m, \quad (11)$$

where:

$$x = \frac{-m(u + \ddot{x}\omega)}{\omega c}. \quad (12)$$

Considering the fact that the mass  $m$  in expression (12) is a constant value, which is due to the design of the CMM, it can be concluded that the displacement of the combined mass has a dependence on the frequency of the oscillations of the base of the CMM -  $\omega$ .

Let's pay attention to the fact that the mass of the base CMM, which is carried out, basically, from a granite monolithic plate, is many times larger than the combined mass, the structural elements of which, in order to reduce the inertial moments, are performed as low as possible with less weight. Taking this into account, we make the assumption that the frequencies of their oscillations do not coincide.

Taking into account the assumption made about the incommensurability of the vibration frequencies of the CMM base and the natural oscillation frequency of the combined mass, the amplitude of the deviation of the measuring head (combined mass) will be equal to the sum of the oscillation amplitudes, ie

$$A_{Br} = \frac{Am\omega^2}{|c - m\omega^2|} \left(1 + \frac{\omega}{f}\right), \quad (13)$$

where:

$A_{Br}$  - is the amplitude of the deviation of the combined mass,  
 $f$  - circular frequency of oscillations of the united mass.

Since  $f = \sqrt{\frac{c}{m}}$ , then equation (13) is to be seen:

$$A_{Br} = \frac{A\omega^2}{f|f - \omega|}. \quad (14)$$

**Conclusions.** The resulting equation (14) makes it possible to estimate the dependence of  $A_{vg}$  on the frequencies with which the oscillations of the base and the combined mass  $f$  take place. It follows directly from this that, at oscillation frequencies ( $\omega \ll f$ ), the contribution of the vibrational perturbation to the value of  $A_{Br}$  is smaller than at frequencies ( $\omega \gg f$ ).

Analyzing the equation for determining the displacement value of the combined mass (13), we conclude that with the increase in the frequency of external vibrational effects, the mass displacement of the combined mass will decrease.

It should be noted that the mathematical dependence between the displacement, speed, acceleration described above is valid only for monochrome signals (sinusoidal), and therefore, in fact, the amplitude of oscillations of the combined mass with respect to the basis of the CMM will have a complicated time implementation, depending on the mutual superimposition of the phases of the vibration accelerations.

Regarding the static calculation: high structural stiffness of applied materials (in many constructions of portal CMM it is granite of dense rocks) allows to significantly reduce static deformations and therefore they are small in size in comparison with the resolution of measurement.

Theoretical studies of the influence of vibration disturbances on the part of the environment on the CMM allow to make calculations and to quickly determine the values of the vibrational effects on the measurement results.

### **References**

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