UDC 621.865.8

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Manipulation robot program trajectories at restrictions in the form of obstracles

The method of synthesis of mechanical trajectories in accord with manipulation robot degrees of freedom is developed. Manipulator segments configurations and obstacles are approximated by semi-spaces that are restricted by plane surfaces. The fact of collision of the robot with obstacles is traced to the objective of linear in equation systems compatibility definition. The algorithm of problem solution on the basis of dynamic programing method is elaborated.

General formulation

Generally the manipulation robot represents open kinematic system consisting of segments and joints connecting them in a successive order [1]. Segments geometrical sizes, joints aspect and limits of their displacement set a working space in any point, where it is possible to position the robot gripper. The joint aspect is defined by design features and manipulation robot drives possibilities. As a rule, they represent kinematic pairs of 5th class and can be set by logic variables p_i:

$$p_i = \begin{cases} 1 & \text{if translational displacement} \\ 0 & \text{if rotational displacement} \end{cases} \qquad (1)$$

Variable quantities defining the gripper position and orientation in working space arejoint coordinates values, q_i=1-2, …, n, in powers of the robot degree of freedom, where n –is the number of degrees of freedom.

Methodology for objects formal description

The objective of robot programmed trajectories synthesis consists in definition of joint coordinates values in powers of degree of freedom, providing gripper displacement along predetermined trajectory with given accuracy, at fulfillment of conditions of segments mutual non-collision between themselves and with obstacles, which are present in technological space, that is in space where segments can move at the robot technological operations performance. As the robot consists of the basis and segments representing geometrical objects that perform movements in technological space containing obstacles, which also represent geometrical objects, it is necessary to develop the methodology of their formal description. For this purpose, the manipulation robot is represented in the form of complex geometrical object consisting of a set of geometrical sub-objects: fixed base and mobile segments, segments…, n-th segment, the location of each of which is defined by joint coordinates values. Each sub-object we is represented in the form of logic expressions. $R_b(x,y,z)$, $R_l^1(x,y,z,q_1)$, $R_l^2(x,y,z,q_1,q_2)$, …,

$R_l^n(x, y, z, q_1, q_2, ..., q_n)$, representing correspondingly: the base, 1-segment, 2-segment..., n-th segment. Obstacles existing in technological space are also represented in the form of a set of fixed geometrical objects described in the form of logic expressions: $R_p^1(x,y,z)$, $R_p^2(x,y,z)$,..., $R_p^m(x,y,z)$, correspondingly, where m - number of obstacles. Logic function describing geometrical objects has the following appearance [3]:

$$R(x, y, z) = R_1 L R_2 L ... L R_N = 1 \qquad (2)$$

R_k, (k=1,2,…N) logic variables defined by following expressions

$$R_k = \begin{cases} 1, & B_k(x, y, z) \le 0, \\ 0 \text{ in the} & \text{contrary case} \end{cases},$$

where $B_k(x, y, z) \le 0$ - is an inequation setting or approximating k part of geometrical object border line; N - quantity of inequations; L - signs of logic operations of conjunction, disjunction or negation. Robot gripper predetermined movement trajectory is approximated by the set of points $A_j(x_j, y_j, z_j)$, j=1,2, …, m, where m - number of points approximating the trajectory. The distance between nearly points is defined from the condition of a trajectory curvature value permanency:

$K = \frac{\alpha}{d_{j,j+1}}$,where α - is the rate of tangent line unitary vector angle change in points $A_j(x_j, y_j, z_j)$ and $A_{j+1}(x_{j+1}, y_{j+1}, z_{j+1})$; K = const, depth of camber coefficient. Let us assume that the robot kinematics configuration defined by the vector $Q^j(q_1^н, q_2^н, ..., q_n^н)^T$, and gripper location in points $A_j(x_j, y_j, z_j)$ in the system of coordinates bundled to the robot fixed base is set. It is necessary to define the vector $Q^{j+1}(q_1^{j+1}, q_2^{j+1}, ..., q_n^{j+1})^T$ providing gripper dislocation to the other point $A_{j+1}(x_{j+1}, y_{j+1}, z_{j+1})$ set in the same system of coordinates.

Description of task of manipulating robot travelling path synthesis

Then the definition of objective of manipulation robot movement trajectories according to the degrees of freedom can be represented in the following form:

It is necessary to minimize (to maximise) the kinematic criteria of type quality [2]:

$$J = \sum_{j=1}^{m-1} \sum_{i=1}^{n} C_i (q_i^н - q_i^{j+1})^2 \to \min(\max) \qquad (3)$$

at restrictions defining a condition of points intromission approximating the trajectory into working space of the handling manipulation robot [3]:

$$\forall A_j(x_j, y_j, z_j), \quad j = 1,2,..., m : D_1 L D_2 L ... L D_M = 1 \qquad (4)$$

Where C_i - is the coefficient characterizing dynamic parameters of the i-degree of freedom drive on in advance set parameter (power inputs, speed, accuracy, etc.);

q_i^j, q_i^{j+1} - are the elements of vectors Q^j and Q^{j+1} accordingly; D_k, (k=1,2, …, M) –are the logic variables defined by following expression:

$$D_k = \begin{cases} 1, B_k(x,y,z) \le 0 \\ 0, in\ the\ contrary \end{cases}$$

where $B_k(x,y,z) \le 0$ - is the in equation setting or approximating k part of working space of the manipulation robot; M - quantity of inequalities; and also the additional restrictions considering possible mutual collisions of robot segments (5) and robot segments with obstacles (6):

$$R_b \wedge R_l^1 \wedge R_l^2 \wedge \cdots \wedge R_l^n = 0 \qquad (5)$$

$$(R_b \vee R_l^1 \vee R_l^2 \cdots \vee R_l^n) \wedge (R_p^1 \vee R_p^2 \vee \cdots \vee R_p^N) = 0 \qquad (6)$$

Where n –is the quantity of manipulator segments, N – is the quantity of obstacles in technological space. For practical implementation of the synthesis algorithm taking into account possible mutual collisions of segments and robot segments with obstacles that are in a working zone the definition of the fact of collision is necessary. For this obstacles that are in the working zone and robot segments are approximated by the polyhedrons, described by systems of linear inequalities of the type (7):

$$\begin{cases} a_{1,1}^k x + a_{1,2}^k y + a_{1,3}^k z + a_{1,4}^k \ge 0, \\ a_{2,1}^k x + a_{2,2}^k y + a_{2,3}^k z + a_{2,4}^k \ge 0, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m,1}^k x + a_{m,2}^k y + a_{m,3}^k z + a_{m,4}^k \ge 0. \end{cases} \qquad (7)$$

where $a_{i,j}^k$, j=1,2,3,4, i=1,2..., m, - is the coefficients setting faces of a polyhedron in OXYZ space approximating k obstacle (segment), m – is the number of this polyhedron faces. Then, mutual collisions of the manipulation robot I obstacle (segment) with k obstacle (segment) it is defined by a condition of existence of solution of the following system of linear inequations:

$$\begin{cases} a_{1,1}^k x + a_{1,2}^k y + a_{1,3}^k z + a_{1,4}^k \ge 0, \\ a_{2,1}^k x + a_{2,2}^k y + a_{2,3}^k z + a_{2,4}^k \ge 0, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m,1}^k x + a_{m,2}^k y + a_{m,3}^k z + a_{m,4}^k \ge 0, \\ a_{1,1}^l x + a_{1,2}^l y + a_{1,3}^l z + a_{1,4}^l \ge 0, \\ a_{2,1}^l x + a_{2,2}^l y + a_{2,3}^l z + a_{2,4}^l \ge 0, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n,1}^l x + a_{n,2}^l y + a_{n,3}^l z + a_{n,4}^l \ge 0. \end{cases} \qquad (8)$$

Using well-known the linear algebra theory methods it is possible to define the fact of handling robot segments collision with present obstacles and robot segments collisions between themselves that result from a condition of compatibility of linear inequations system (8), [4]. One of the ways of the problem solution is scaling of all possible minors of 3 linear system inequations (8). If there is a minor other than zero, then systems (8) are not compatible, that is there is no fact of segment collision between themselves and with obstacles that are in the working zone. The same problem can be solved by matrix theory methods, on the basis of Rouché–Capelli theorem [4]. Initial systems of inequations (8) by the addition of unrestricted variables are reduced to the linear equations systems. The coefficient matrix rank and expanded matrix rank is computed. At match condition the system has at least one solution, otherwise solutions do not exist.

Algorithm of programmed trajectories synthesis

Program trajectories synthesis algorithm in accord with handling robot degrees of freedom taking into account robot segments mutual collisions and with present obstacles in the working zone substantially consists of two stages. At the first stage of algorithm the weighed graphs, the apexes of which are possible values q_i is constructed, and ribs correspond to kinematic pairs. At the second stage on the basis of Bellmana dynamic programing method there is a set $Q(q_{i,j})$ that minimizes performance criterion of the type (3). Definition of q_i possible values with digitization step depends on the robot kinematic pair type that is on values of parameter p_i, p_{i+1}.

Let's describe parts of working space forming by the segments movement since r-degree of freedom of manipulation robot in the form of logic expression:

$$L_r = D_1 L D_2 L \ldots L D_m = 1 \qquad (9)$$

where D_k, (k=1,2, …, m) –are the logic variables setting or approximating a part of handling robot working space.

Expression (9) is obtained as follows: using values of segments lengths l_i, i=1,2, …, n; joints parameters, and also considering constructive restrictions imposed on joint coordinates values changes:

$$q_i^H \le q_i \le q_i^B \qquad (10)$$

where q_i^H, q_i^B - are the lower and upper value of joint coordinate i -th degree of freedom the manipulator, it is necessary to represent a part of robot working space graphically. Obtained graphic representation is used for definition of elementary surfaces restricting a part of working space that are described by logic variables D_k. Bundling variables D_k according to the obtained graphical configuration of a part of working space, the expression of the type is obtained (9). Let us suppose that the initial configuration of the kinematic scheme providing the fulfillment of the condition of robot segments non-collision between them and with the obstacles that are in technological condition performance space is set.

The algorithm is constructed on the basis of the analysis of the ways in the weighed graph containing all possible solutions and has the following type:

Step 1.Input of initial values: the coordinates of points approximating gripper mechanical trajectory

$A_j(x_j, y_j, z_j)$, $j = 1,2,..., m$; the logic expressions $L_1, L_2,.., L_{n-2}$ presenting working space and subspaces, logic expressions $R_p^1(x, y, z)$, $R_p^2(x, y, z)$,..., $R_p^m(x, y, z)$, $R_b(x, y, z)$, $R_l^1(x, y, z, q_1)$, $R_l^1(x, y, z, q_1, q_2)$, ..., $R_l^1(x, y, z, q_1, q_2,..., q_n)$ describing robot obstacles, base and segments as geometrical objects, $\Delta q_1, \Delta q_2,..., \Delta q_n$ solution search steps values according to degrees of freedom $Q^н(q_1^н, q_2^н,..., q_n^н)^T$, $Q^в(q_1^в, q_2^в,..., q_n^в)^T$ - vectors setting lower and upper values of joint coordinates values change, initial location of handling robot configuration $Q^0(q_1^0, q_2^0,..., q_n^0)^T$.

Step 2. If the condition is fulfilled,

$\forall A_j(x_j, y_j, z_j)$, $j = 1,2,..., m$: $L_1 = 1$

then is to pass to a step 3, otherwise the problem is meaningless and the solution comes to the end.

Step 3. $j = 1$ **Step 4**. Under each degree of freedom the joint coordinates change interval is divided intonequal intervals. As a result of such procedure the oriented graph of possible configurations will be obtained. Let us set this graph by an adjacency triangular matrix of (nm+2 $\times$ nm+2)dimensions, where n–is the number of robot degrees of freedom, m–is the quantity of joints fulfilling the condition (12), the elements of adjacency matrix are defined on the basis of following expression:

$$a_{i,j}^{i+1,j} = \begin{cases} 1, \text{if the } q_{i,j} \text{ apex, is adjacent to the } q_{i+1,j} \text{ apex,} \\ 0, \text{in the contrary.} \end{cases}$$

Step 5. For each possible handling robot configuration the condition performance is checked $R_b \wedge R_l^1 \wedge R_l^2 \wedge \cdots \wedge R_l^n = 0$, then the value $a_{i,j}^{i+1,j} = 1$ in the contrary case $a_{i,j}^{i+1,j} = 0$.

Step 6. For each possible handling robot configuration the condition performance is checked в$(R_b \vee R_l^1 \vee R_l^2 \cdots \vee R_l^n) \wedge (R_p^1 \vee R_p^2 \vee \cdots \vee R_p^m) = 0$, then the value $a_{i,j}^{i+1,j} = 1$, in the contrary case $a_{i,j}^{i+1,j} = 0$.

Step 7. Further, the values are $q_{i,j}$ re-defined on the basis of the following expression are:

$$q_{i,j} = \min \sum_{j=1}^{m} C_i (q_i^0 - q_{i,j})^2 .$$

Step 8. For final apex of the graph we get:

$$q^{K} = \min \sum_{j=1}^{m} C_n (q_n^0 - q_{n,j})^2$$

Step 9. $j = j + 1$

Step 10. If, $j \le n$ is to pass to step 4, otherwise to pass to step 11.

Step 11. Joint coordinates values $Q(q_{i,j})$.

Further, the problem is solved under the known scheme [1,2]. On the basis of inverse problem obtained solution under a location with application of spline functions method in the space of joint coordinates it is possible to obtain a trajectory providing matching of the gripper with program motion in points approximating mechanical trajectory. Further, considering restrictions on speeds and accelerations, we gain the control program of the manipulation robot.

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