Quasi-linear viscoelastic model for randomly reinforced magnetorheological elastomers

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Abstract. Combined numerical and analytical determination of overall dynamic response and creep behavior of random multi-component reinforced elastomers are proposed. Magnetorheological elastomers are considered here as an example of smart materials, composed of micro-sized magnetic carbonyl iron particles dispersed in a non-magnetic silicone rubber. The viscoelastic behavior of rubber matrix is described by Rabotnov's type quasi-linear law. The random structure of composite analyzed, so constitutive equations for statistical fluctuations of first and second order displacement, nonlinear Green deformation, nominal or Cauchy stress in the representative volume are used. Upon application of the integral Laplace-Carson and Fourier transforms, the boundary value problem for the local stress and strain fields becomes similar to a nonlinear elastic one. The volume concentration of carbonyl iron remains unchanged after transforming from the time domain to the Laplace-Carson domain, as in the case of non-aged materials. The explicit determination of the inverse transform is not straightforward, and numerical methods are required. Efficient algorithms for numerically evaluating the inverse Laplace transform we use here from NAG-Fortran library. Numerical experiments by finite elements modelling were carried out with the aim of choosing the optimal structure and composition of multi-component magnetorheological elastomer materials.

1. Introduction

The high-performance composite materials are widely used in aviation, especially over the last decades [1, 2]. The rather new type of this material is represented by reinforced elastomers [3-5]. The mechanical behavior of these materials is characterized by creep, change in strain over time at constant stress, relaxation of stress, change in stress over time at constant strain, damping of free vibrations, etc. Elastomers may have highly elastic deformations, which are characterized by a larger value in comparison with elastic deformations of solids. Moreover, highly elastic deformations are as reversible as elastic ones. The most important feature of elastomers is high deformability without fracture.

Many elastomers are incompressible [6, 7]. The main relaxation properties are creep, stress relaxation, recovery or reverse creep. General deformation consists of two components - elastic and viscoelastic. Composites based on elastomeric matrix are used for effective vibration damping. The wide distribution of elastomeric materials is associated with their unique properties. Elastomeric parts increase the reliability and durability of structures, and reduce their material and energy consumption.

Magnetorheological elastomers (MREs) are considered as smart composites [8, 9]. Such materials are composed of micro-sized magnetic particles dispersed in a non-magnetic elastomeric matrix. MREs have attracted great interest because their mechanical and rheological properties can be controlled by the application of an external magnetic field, due to magneto-rheological effect [10, 11]. These composites have been made from various types of matrix materials, such as silicone rubber and thermoplastic elastomers [4]. Micro-sized carbonyl iron particles are the most common type of fillers [5, 9].

Such materials inherit the main properties of the elastomeric matrix, such as large deformations, stress softening effect, amplitude and frequency dependency, reduction of stiffness at cyclic loading. So the dynamic mechanical properties of MREs are studied by the equations for viscoelastic materials. Fractional derivative viscoelastic models have been proposed in [6, 12] and used here to study the viscoelastic behavior of elastomers [13-15].

Change in the properties of MREs under the application of an external magnetic field is most extensively studied in recent years. A Rabotnov's four-parameter fractional derivative model was applied to simulate the viscoelastic behavior of the isotropic materials in [12-17]. The dependence of dynamic moduli and loss factor on the frequency and magnetic flux density was numerically calculated using this model [13, 18].

2. Materials and Methods

Viscoelastic models based on fractional derivatives include fractional derivative terms, added to viscous terms and elastic terms. It can be used in both the time and frequency domain [6, 7]. Methods of rheological parameters determination presented in [12, 16]. The constitutive equation for the four-parameter fractional derivative model in the time domain may be written as follows [7, 13]:

$$\tau^m \frac{d^m\sigma}{dt^m} + \sigma(t) = \mu_e \tau^m \frac{d^m e}{dt^m} + \mu_r e(t), \tag{1}$$

where σ is stress, e is deformation, μ_r is the static, relaxed, elastic shear modulus, $\mu_e = \mu_r + \mu_v$ is the high frequency limit value of the dynamic modulus, τ is the relaxation time, and m is the fractional parameter with value varying between 0 and 1 [7, 15].

Representing the model in the frequency domain is more useful and much easier than that in the time domain [1, 3]. Therefore, equation (1) is represented in the frequency domain using Fourier transform as follows:

$$\sigma(\omega)(1+(i\tau\omega)^m) = e(\omega)[\mu_r(1+(i\tau\omega)^m) + \mu_v(i\tau\omega)^m]; \quad \mu_v = (\mu_e - \mu_r). \tag{2}$$

Parameters of the fractional derivative model for isotropic MREs under different flux densities are fitted in [4, 5] $\mu_r = 0.368\,MPa$; $\mu_e = 1.686\,MPa$; $\tau = 0.009\,s \quad m = 0.281\,rad$.

Complex shear modulus of the four-parameter fractional derivative viscoelastic model in the frequency domain takes the form of

$$\mu(i\omega) = \mu' + i\mu'' = \frac{\mu_r + \mu_e(i\tau\omega)^m}{1+(i\tau\omega)^m} \tag{3}$$

The complex shear modulus $\mu(i\omega)$ consists of a real part $\mu'(\omega)$, the storage modulus or the rigidity, which characterizes the stiffness of the viscoelastic material, and an imaginary part $\mu''(\omega)$, called loss modulus or the energy dissipation, which characterizes the viscous behavior. The expressions for the storage and loss modulus obtained for the model we use here are as:

$$\mu' = \frac{\mu_r[1+z\cos(y)] + \mu_e z[\cos(y)+z]}{\Delta}; \ \ \mu'' = \frac{\mu_v z\sin(y)}{\Delta}; \ \Delta = 1+2z\cos(y)+z.$$

Therefore, the loss tangent in the frequency domain can be expressed as:

$$\delta(\omega) = \frac{z\mu_v sin(y)}{\mu_r[1+zcos(y)]+\mu_e[cos(y)]+z}, \quad z=(\tau\omega)^m, \ y=\frac{\pi m}{2}, \tag{4}$$

Visco-elastomeric materials are often used as noise-vibration isolators in aviation, mechanical engineering etc. We use here a quasilinear fractional derivative Rabotnov's type viscoelastic model [7, 14, 18] to describe the static creep and dynamic shear behaviour of randomly reinforced rheological elastomers as a function of the matrix and particle content. Due to the viscoelastic matrix, the predominant behaviour of MREs is the viscoelastic one.

2.1. Quasi-linear viscoelasticity

The starting point for the development of the stress strain relation is the general form of the constitutive relation for viscoelastic elastomers, given by [11, 15]

$$\sigma_{ij} = -p\delta_{ij} + F_{i\alpha}\phi_{\alpha\beta}\{(e_{\alpha\beta}(t-s), e_{\alpha\beta}(t))\}_{s=0}^{\infty} F_{j\beta} \tag{5}$$

where σ_{ij} is the Cauchy stress, p is the pressure, $F_{i\beta}$ denotes the displacement gradients, $\{\phi_{\alpha\beta}\}_{s=0}^{\infty}$ is a tensor-valued functional with a dependence not only on the strain history but also on the current strain. To analyze the problems of prediction deformations within the framework of the hereditary creep model, we use the modified Shapery principle of correspondence [7, 14, 17]. We will use the notation adopted in [6, 13]. We suppose the existence of a function of stored energy $W(\mathbf{e})$ and additional energy $U(\boldsymbol{\sigma})$, which make it possible to find instantaneous deformations $\mathbf{e}(t)$ during creep, or instantaneous stresses $\boldsymbol{\sigma}(t)$ in relaxation process

$$\mathbf{e}^e(t) = \frac{\partial U}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}, \mathbf{x}, t); \qquad \boldsymbol{\sigma}^e(t) = \frac{\partial W}{\partial \mathbf{e}}(\mathbf{e}, \mathbf{x}, t). \tag{6}$$

Here, stresses and strains are referred to the orthogonal Cartesian coordinate system, and the presence of coordinates and time in the list of arguments of the function indicates the possibility of its being used for the analysis of inhomogeneous media, composite materials, and in problems where the time factor is essential. We consider the material in conditions of steady creep, and write the general governing equations of hereditary creep in the form

$$\frac{\partial W}{\partial \mathbf{e}}(\mathbf{e}, \mathbf{x}, t) = \boldsymbol{\sigma}^R(t) = \int_{-\infty}^{t} \hat{\mathbf{J}}(t-s)d\boldsymbol{\sigma}(s) = (\hat{\mathbf{J}} * d\boldsymbol{\sigma})(t), \quad \hat{\mathbf{J}}(t) = \mathbf{J}(t)/\mathbf{J}(0), \tag{7}$$

where $\boldsymbol{\sigma}(t)$ is the Cauchy stress tensor. Stress $\boldsymbol{\sigma}^R(t)$ is so called modified stress [7, 18], restored from the known current values. Relations (6) are the governing equations of a nonlinear elastic body. In order to further use the principle of viscoelastic correspondence, we write down the relations between current deformations and current stresses in the form of the governing equations of hereditary creep, developed in [12, 13]. The hereditary integrals used here are linear functionals with creep and relaxation functions depending on the spatial coordinate. This allows us to consider the differentiation with respect to the coordinate and hereditary integration over time as permutation operations.

2.2. Multi-component random composite theory

Let us consider some representative volume V_R of the composite material in the undeformed state and introduce Cartesian coordinate system x^A with orts $\mathbf{e}_A$ in the reference configuration k_R. The position of the material point in this configuration is determined by the radius vector $\mathbf{x} = x^A\mathbf{e}_A$. As a result of creep deformation, the position of a material particle in space changes with time and in the current configuration $k = k\left(k_R, t\right)$ at the time t is given by a vector with radius

$$\mathbf{y} = \mathbf{y}(\mathbf{x}, t) = y^m\mathbf{e}_m, \qquad \mathbf{y} = \mathbf{x} + \mathbf{u}(\mathbf{x}, t), \tag{8}$$

where $\mathbf{y}(\mathbf{x}, t)$ is a continuous function of two variables; $\mathbf{u}(\mathbf{x}, t)$ - displacements vector. The following kinematic relations take place [6, 13]

$$\mathbf{y} = \mathbf{F}\mathbf{x}, \qquad \mathbf{F} = \mathbf{1} + \mathbf{H}, \qquad 2\mathbf{e} = \mathbf{H} + \mathbf{H}^T + \mathbf{H}^T\mathbf{H}, \qquad \mathbf{H} = (\nabla\mathbf{u})^T. \tag{9}$$

Here $\mathbf{F}$ is the direct deformation gradient; $\mathbf{H}$ gradient displacements; $\mathbf{e}$ is Green deformation tensor; $\mathbf{1}$ is unit second order tensor; ∇ is gradient operator in the coordinates of the reference configuration.

Then, stress vector $\mathbf{t}_N$ is introduced in the deformed material at the site which in the reference configuration is determined by the ort $\mathbf{N}$. The stress vector $\mathbf{t}_N$ is related to a unit area of the undeformed state. In this case, we can write

$$\mathbf{t}_N = N_A T_{Am}\mathbf{e}_m. \tag{10}$$

Here T_{Am} is the asymmetric tensor of nominal stress (Piola-Kirchhoff) [1, 6]. The true Cauchy stress tensor σ_{ij} is related to the nominal stress tensor T_{Am} by the relations

$$\sigma_{im} = J^{-1}F_{iA}T_{Am}, \qquad J = \det\|\mathbf{F}\| . \tag{11}$$

Elastomers are usually considered as viscoelastic incompressible material [3, 4], so we choose to describe its elastic properties potential in the form of Mooney-Rivlin [6, 7]

$$W = C_1(\mathrm{I}_1 - 3) + C_2(\mathrm{I}_2 - 3). \tag{12}$$

Here $\mathrm{I}_1, \mathrm{I}_2$ are the invariants of the strain measure $\mathbf{B}$,

$$\mathrm{I}_1 = tr(\mathbf{B}), \qquad \mathrm{I}_2 = \frac{1}{2}\left(\mathrm{I}_1^2 - tr(\mathbf{B}^2)\right), \qquad \mathbf{B} = \mathbf{F}^T\mathbf{F} , \tag{13}$$

C_1 , C_2 are elastic constants of material $C_1 = \mu(\frac{1}{2}+\beta)$; $C_2 = -\mu(\frac{1}{2}-\beta)$, tr is tensor convolution operator [2, 6]. Up to terms of the second order of smallness, by displacements we have

$$\boldsymbol{\sigma} / \mu = -\tilde{p}\mathbf{1} + 2\mathbf{e} + \mathbf{H}\mathbf{H}^T - 4(\frac{1}{2}-\beta)\mathbf{e}^2 . \tag{14}$$

In (14) $\tilde{p}$ is the indefinite dimensionless constant, and β is the dimensionless modulus of the material. The incompressibility condition in the theory of viscoelasticity of the second order is written [13] as

$$tr(\mathbf{H}\mathbf{H}^T) + A_2(\mathbf{e}) = 0 . \tag{15}$$

Physical relationships with accuracy up to second order are represented in the form

$$\boldsymbol{\sigma} = -p\mathbf{1} + \mu(\frac{1}{2}+\beta)\mathbf{B} - \mu(\frac{1}{2}-\beta)\mathbf{B}^{-1}, \tag{16}$$

where μ and β are constants.

After averaging over undeformed representative volume, nominal stress tensor can be used as a macroscopic variable of the nonlinear theory of elasticity [1, 2] and viscoelasticity. Given this, the equations of state of the first linear and second nonlinear approximations of the nonlinear theory of elasticity are written as:

$$\begin{aligned} \mathbf{T}_{(1)} &= \boldsymbol{\sigma}_{(1)} = p_{(1)}\mathbf{1} + 2\mu\mathbf{e}_{(1)}, \\ \mathbf{T}_{(2)} &= p_{(2)}\mathbf{1} + 2\mu\mathbf{e}_{(2)} - p_{(1)}\mathbf{H}_{(1)} - \mu\mathbf{H}_{(1)}^2 - \gamma\mathbf{e}_{(1)}^2. \end{aligned} \tag{17}$$

Here, $p_{(1)}$, $p_{(2)}$ are scalars, and subscript in parentheses means the order of non-linear approximation,

$$\mu = 2(C_1 - C_2), \qquad \gamma = 8C_2, \qquad 2\mathbf{e}_{(1)} = \mathbf{H} + \mathbf{H}^T, \tag{18}$$

where μ is the shear modulus; γ is instant modulus of the second-order nonlinear theory.

2.3. Multi-component materials

In the case of multi-component materials, we use the design scheme proposed in [8], the essence of which is as follows. If a theoretically exact solution is known

$$\mathbf{e}_i = \mathbf{G}_i\mathbf{e}_m, \tag{19}$$

then the deformation concentration tensors A_i, A_m [18] are determined by the expressions

$$\mathbf{A}_i = \mathbf{G}_i\mathbf{A}_m; \qquad \mathbf{A}_m = \left(c_m\mathbf{I} + \sum_i c_i\mathbf{G}_i \right)^{-1} . \tag{20}$$

Then an approximate solution is constructed by replacing the tensor G_i, unknown in the general case, with the tensor T_i, known from relations, which connects the mean deformations of the inclusions marked with a number and the mean deformations of the representative volume, i.e.

$$\mathbf{e}_i = \mathbf{T}_i \mathbf{e} \tag{21}$$

Then for tensors $\mathbf{A}_i$, $\mathbf{A}_m$, we obtain the representation

$$\mathbf{A}_i = \mathbf{T}_i \mathbf{A}_m; \qquad \mathbf{A}_m = \left(c_m \mathbf{I} + \sum_i c_i \mathbf{T}_i \right)^{-1} \tag{22}$$

In this paper, we define the tensor $\mathbf{G}_i$ from the solution obtained for an incompressible composite material in [8], therefore we take

$$\begin{aligned} &G_i = \mathrm{T}_i = 1 + z_i f_i; \qquad z_i (1 - g_i f_i)^{-1} g_i; \\ &f_i = \mu_i - \mu_m; \qquad z_i \left\langle z_i^n \right\rangle \Omega. \end{aligned} \tag{23}$$

Here, the matrix was obtained by analyzing the stress-strain state in inclusions with a number i. Thus, to determine tensors, you can write the following expressions

$$\begin{aligned} &\mathrm{A}_i = A_m (1 + z_i f_i); \\ &\mathrm{A}_m = \left(1 + \sum_i c_i z_i f_i \right)^{-1}. \end{aligned} \tag{24}$$

By direct verification, one can be seen that in the case of a two-component material, representation (24) is identical to formulas for two component material. From this we can conclude, in particular, that the accuracy of the results (24) corresponds to the level of well-known solutions in the mechanics of inhomogeneous materials [1, 18].

2.4. Second order nonlinear solution

Equilibrium equations of a representative volume written relative to the random fluctuations of second-order displacement have the form

$$\nabla \mathbf{C} \nabla \mathbf{u}'_{(2)} = -\nabla \boldsymbol{\tau}_{(2)}; \quad \boldsymbol{\tau}_{(2)} = \mathbf{f}\mathbf{e}_{(2)} - \mu \mathbf{H}^2_{(1)} - \gamma \mathbf{e}^2_{(1)}. \tag{25}$$

The gradients of displacements of the second approximation are decomposed into dilatational and deviator parts. Then, for the deviators of the second approximation the condition of incompressibility will coincide in shape with the analogous condition of the linear theory of viscoelasticity [15, 17]. The field of displacements, the field of deformations and stresses must be supplemented with the following relations

$$\begin{aligned} &\mathbf{e}(\mathbf{x},t) = \frac{1}{2}\left(\mathbf{H}(\mathbf{x},t) + \mathbf{H}^T(\mathbf{x},t) \right); \quad Div\, \boldsymbol{\sigma}(\mathbf{x},t) = 0; \\ &\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}(\mathbf{x})\mathbf{e}(\mathbf{x}) + \mathbf{e}(\mathbf{x})\mathbf{D}(\mathbf{x})\mathbf{e}(\mathbf{x}); \quad \mathbf{u}(\mathbf{x},t) = \bar{\mathbf{e}}(\mathbf{x},t) \cdot \mathbf{x} + \mathbf{u}'(\mathbf{x}), \end{aligned} \tag{26}$$

In problems with cyclic loading, the solution can be obtained by Fourier transforms [2]. This makes it possible to use the Green function $\boldsymbol{G}$ of a linear solution of the first order, and obtain an integral equation to determine the deviators of displacement gradients

$$\mathbf{H}_{(2)} = \mathbf{G} * \left(\mathbf{fe}_{(2)} - \mu \mathbf{H}_{(1)}^2 - \gamma \mathbf{e}_{(1)}^2 \right) + \bar{\mathbf{H}}_{(2)} . \tag{27}$$

We average this equation, provided that the coordination argument of the left side is in the volume occupied by spheroidal inclusions with known properties and oriented in the n-direction [8]. Finally, the relations between random parts of $\mathbf{H}_{(2)}$ simultaneously with the constitutive elasticity equation in Fourier space are obtained. At the same time, nonlinear terms are expressed through macroscopic deformations of the representative volume of the composite material

$$\begin{aligned} &\mathbf{H}_{r(2)} = m_{Ar}^2 \mathbf{e}_{(1)}^2 + m_{Ar} (\mathbf{ew} + \mathbf{we}) + \mathbf{w}^2; \\ &\mathbf{e}_{r(2)} = m_{Ar}^2 \mathbf{e}_{(1)}^2; \qquad 2\mathbf{w} = \mathbf{H} - \mathbf{H}^T . \end{aligned} \tag{28}$$

Substituting this solution into the second-order averaged physical relations from (26), we find the constitutive law of connection between the macroscopic Piola-Kirchhoff stresses and the gradients of displacements of the second approximation

$$\mathbf{T}_{(2)} = p_{(2)} \mathbf{1} + 2\tilde{\mu} \mathbf{e}_{(2)} - \left(p\mathbf{H} + \tilde{\mu} \mathbf{H}^2 + \tilde{\gamma} \mathbf{e}^2 \right)_{(1)} . \tag{29}$$

Overall coefficients $\tilde{\mu}$, $\tilde{\gamma}$ are calculated as

$$\tilde{\mu} = \sum_{r=1}^{R} c_r \mu_r m_{Ar}; \qquad \tilde{\gamma} = \sum_{r=1}^{R} c_r \mu_{Ar}^3 (\mu + \gamma)_r - \tilde{\mu} , \tag{30}$$

i.e. m_{Ar} is components of matrices $\mathbf{A}_r$ are defined in [8, 18].

2.5. Examples

As an example, consider a composite material which is a viscoelastic matrix based on resin ED-6, reinforced with two types of inclusions (SiC and ferromagnetic steel short fibers). Nonlinear elastic properties of inclusions and matrix are presented in Table 1.

Table 1. Nonlinear elastic material constants, GPa, for the Ferromagnetic Steel/SiC/Epoxy composite.

Material	E, GPa	ν	ν_1, GPa	ν_2, GPa	ν_3, GPa
Steel	207.4	0.27	-1650.0	-309.0	-200.0
SiC	440.3	0.171	-227.2	31.5	-170.75
Epoxy	3.15	0.382	13.3	4.09	-10.02

In the case of model for composite with incompressible matrix we use such parameters [1, 8]

$$\begin{aligned} &\mu_1 = 10,3 \text{ MPa}, \qquad \gamma_1 = 62{,}18 \text{ GPa}, \qquad \mu_2 = 3{,}8 \text{ GPa}, \\ &\gamma_2 = 4,72 \text{ GPa}, \qquad \mu_m = 1,686 \text{ MPa}, \qquad \gamma_m = 2,2 \text{ MPa}. \end{aligned}$$

In the calculations, it was assumed that the relative size of inclusions of the first type (micro-sized carbonyl iron particles) is $r = 10$, and inclusions of the second type (SiC) are assumed to be spherical $r = 1$. Elastomeric matrix has shear viscoelastic properties presented in Table 2.

3. Results

The above relations of nonlinear viscoelasticity of Rabotnov type make it possible to describe the phenomena of propagation of harmonic waves. Indeed, the generalized Hooke's law in the case of viscoelastic deformation has the form

$$\sigma_{ij} = C^{*}_{ijkl} e_{kl} , \tag{31}$$

where C^{*}_{ijkl} is the tensor of linear viscoelastic operators such that

$$C^{*}_{ijkl} e_{kl} = C^{e}_{ijkl} \left[e_{kl}(t) - \int_0^t R_\alpha (t-s) e_{kl}(s) ds \right] \tag{32}$$

Here C^{e}_{ijkl} is the tensor of instant elastic moduli; $R_\alpha(t)$ is the fractional exponent kernel of the viscoelastic aftereffect [12, 14].

As shown in [7, 14], finding the macroscopic operators describing the relationship between average stresses $\bar{\sigma}_{ij}(t)$ and strains $\bar{e}_{ij}(t)$ is reduced to solving equations similar to the nonlinear elastic problem (25), (26), where the corresponding tensor of operators C^{*}_{ijkl} should be substituted instead of the tensor of elastic moduli C_{ijkl}. Due to the fact that the operation of integration over time is interchangeable with integration over coordinates and averaging over the volume to solve the problem in a viscoelastic formulation, one can use the corresponding solution of its elastic analogue and replace the elastic moduli with viscoelastic operators (32) only in the final expressions. Thus, the Volterra principle can be used to determine the reduced viscoelastic properties of a material. For this reason, it is sufficient to substitute the corresponding integral operators in the formulas of the reduced elastic moduli and to decipher the obtained expressions using the algebra of fractional-exponential operators [7, 16].

When modeling composite elastomers, we assume that the filler is elastically deformed, and the binder has shear viscoelastic properties [3, 4]. Experiments carried out at the Institute of Mechanics, and known from literature [1, 6] showed that a SiC filler and steel fibers can be considered as nonlinear elastic. Shear creep deformations at a temperature of 20°C are fairly well described by the linear integral operator

$$\mu^{*} = \mu \left[1 - \xi R^{*}_{\alpha} (\beta t^{\alpha+1}) \right], \tag{33}$$

where $R_\alpha(\beta t^{\alpha+1})$ is the fractional exponential function of Rabotnov [12].

Table 2. The shear elasticity and parameters of viscoelastic Rabotnov's kernel for silicone rubber [5].

flux $B, Tesla$	μ, MPa	ξ	α	β
0.0	1.686	2.937	-0.719	0.820
0.651	2.034	2.764	-0.702	0.732

The parameters of viscoelastic Rabotnov's kernel for epoxy rubber are identified in [12] $\xi = 0.054\ hour^{-0.5}$, $\alpha = -0.5$, $\beta = 0.1764\ hour^{-0.5}$. The fractional exponential Rabotnov operator with kernel $R_\alpha(\beta t^{\alpha+1})$ belongs to the class of well-studied resolvent operators. It possesses a number of properties used in decoding operator expressions [16]. In the cases where it is required to know the exact result of the operator's action on a constant or variable value, we will use the software package in the Fortran F90 shell [13, 18].

3.1. Dispersion and attenuation of harmonic waves in viscoelastic composite elastomers

Using the correspondence principle [7, 14], let us analyze the dynamic problem for an isotropic randomly reinforced elastomer under a stationary cyclic loading regime. If a periodic disturbance with frequency ω acts on a viscoelastic material, then in (31) it is expedient to go over to the complex values

$$\mu(\omega) = \mu_R(\omega) + i\mu_I(\omega), \tag{34}$$

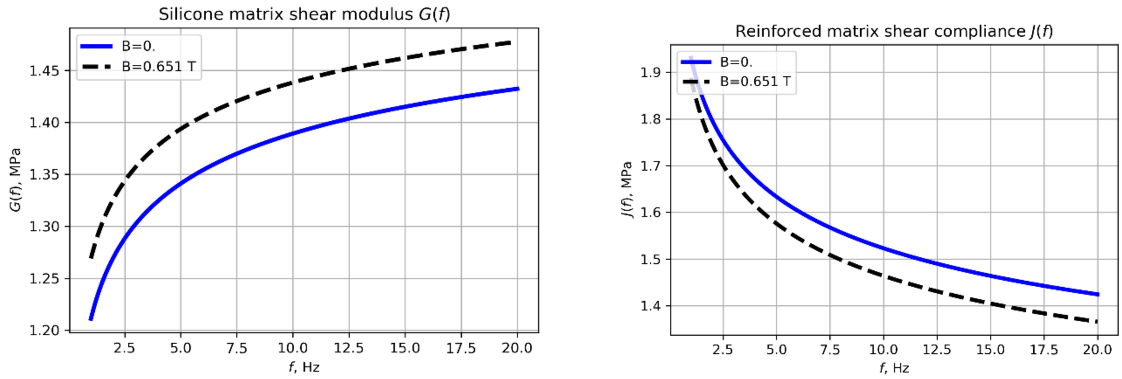
where

$$\mu_R(\omega) = \mu_e \left[1 - \xi\beta^{-1} \frac{z\sin\frac{\pi\alpha}{2} + z^2}{1 + 2z\sin\frac{\pi\alpha}{2} + z^2} \right]; \quad \mu_I(\omega) = \mu_e \left[1 - \xi\beta^{-1} \frac{z\cos\frac{\pi\alpha}{2}}{1 + 2z\sin\frac{\pi\alpha}{2} + z^2} \right], \tag{35}$$

ω is the circular frequency of harmonic vibrations.

3.2. Shear relaxation and shear creep function

The dependence of dynamic properties of the isotropic randomly reinforced silicone matrix on the frequency and magnetic field intensity was numerically modeled. The four-parameter fractional derivative quasi-linear viscoelastic Rabotnov's type model was used. Shear relaxation function $G(f)$, and shear creep function $J(f)$ of composite reinforced with micro-sized carbonyl iron particles were calculated. Results obtained presented on fig. 1. It may be noted that the dependence these functions on stress loading frequency f for external magnetic flux B is really significant as to practical problems. We can also note the agreement of the results presented with the experimental data obtained in [5] in the studied frequency band for different magnetic flux densities.



(a) (b)

Figure 1. Silicone rubber viscoelastic matrix shear relaxation function $G(f)$ (a), and shear creep function $J(f)$ of composite reinforced with micro-sized carbonyl iron particles (b) dependence on stress loading frequency f for different external magnetic flux B.

4. Discussion

Next, we apply the correspondence principle using the Laplace-Carson transform [7, 14]. Thus, the reduced complex characteristics are obtained by replacing the elastic constants in expressions (30) with complex quantities of the form (34), (35). It should also be noted that with rather complex analytical dependences, the numerical methods of the Laplace transform make it possible [18] to obtain the values of the reduced complex characteristics without additional restrictions on the value of viscoelastic damping.

Let us consider as an example a composite of a granular structure, the shear modulus of which is expressed in terms of the constants of the components by the formula from (30)

$$\tilde{\mu} = \sum_{r=1}^{R} c_r \mu_r m_{Ar}. \tag{36}$$

Replacing here the modulus μ_m by its complex value, which depends on the frequency of the cyclic action, we find the reduced viscoelastic characteristic

$$\tilde{\mu}(\omega) = \tilde{\mu}_R(\omega) + i\tilde{\mu}_I(\omega) \tag{37}$$

Let a harmonic wave propagate in an unlimited granular medium that models the reinforced elastomer sample. Then, according to the principle of correspondence [7], the speed of this wave is represented by the formula

$$c_{\mathrm{T}} = \left[\tilde{\mu}_R(\omega) + i\tilde{\mu}_I(\omega)\right]^{\frac{1}{2}} \rho^{-\frac{1}{2}} = \left(\tilde{\mu}_R/\rho\right)^{\frac{1}{2}} \left[1 + i\tilde{\mu}_I(\omega)/\tilde{\mu}_R(\omega)\right]^{\frac{1}{2}} \tag{38}$$

From relations (37) we find the characteristics of the dispersion and damping of a macroscopic wave in a viscoelastic composite medium $\delta(\omega) = \tilde{\mu}_{\mathrm{I}}(\omega)/\tilde{\mu}_R(\omega)$. The change in the attenuation coefficient in granular rheological elastomer depends on the concentration of micro-sized carbonyl iron particles. In this regard, it may be concluded that at low filler concentrations the phenomena of dispersion, damping, and energy dissipation are due to both the viscoelastic properties of the binder and the structural inhomogeneity of the medium. With an increase excitation frequency and magnetic flux density in the concentration of the elastic filler, the predominant effect is the scattering and damping of harmonic vibrations on micro-inhomogeneities of the medium.

Conclusions

The combined analytical and numerical method of prediction of dynamic mechanical properties for randomly reinforced isotropic elastomers made of silicone matrix and micro-sized carbonyl iron particles is presented in this paper. Dynamic mechanical properties are calculated at various frequencies under different magnetic flux densities. The stiffness and damping properties increased with increasing of excitation frequency and magnetic flux density. In the further numerical experiments may be carry out with the aim of choosing the optimal structure and composition of materials for the possible control of frequency, damping properties and long-term strength under given technological constraints.

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