Limit Cycles in Nonlinear Stabilization Systems

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Abstract. An automatic control system is considered to be non-linear if it includes one or more substantially non-linear elements. A feature of nonlinear systems is the possibility of occurrence in them of limit cycles - continuous oscillations. In general, the limit cycle is not sinusoidal. In this case, the amplitude of continuous oscillations does not depend on the external influence and on the initial conditions. The forced oscillation frequency for a periodic input action in a nonlinear system can be either subharmonic or harmonic of the input signal. A feature of nonlinear systems should be considered the phenomenon of jump resonance. The problems of analysis and synthesis of such systems are much more complicated than similar problems for linear systems - there are no universal analytical methods for studying nonlinear systems. In the article the conditions of appearing of periodic undamped oscillations in nonlinear stabilization systems have been considered. Shown, that the introduction into the nonlinear system the corrective circuits will eliminate the possibility of occurrence of limit cycle in the system.

1. Introduction

Many systems contain elements, which are described by nonlinear equations and have essentially nonlinear characteristics. Examples are the elements with characteristics such as insensibility zone, saturation (limitation), the ideal relay, the hysteresis loop, the relay with hysteresis, etc. The system, which includes at least one such element is a nonlinear. Figure 1 shows a block diagram of a nonlinear stabilization system.

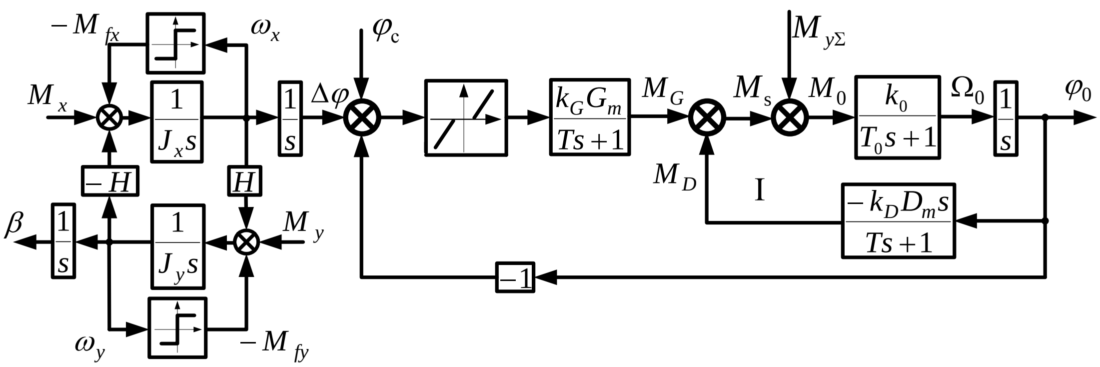

Figure 1. Block diagram of nonlinear system of stabilization.

2. Problem statement

The stabilization moment is formed through the sensor channel of angular deviation and through the speed sensor of angular deviation of control object

$$\overline{M}_s = \overline{M}_G + \overline{M}_D \equiv k_G G_m + k_D D_m,$$

where G_m, D_m are structural stiffness and damping of the system, respectively; $1 \le k_G \le 0$ and $1 \le k_D \le 0$ are coefficients of regulating the stiffness and damping. The system regulator has a linear characteristic with insensibility zone. The friction in the frame pillar of gyroscopic angle sensor is a "dry"

$$M_{fx} = -M_{fm} \operatorname{sign} \omega_x;$$
$$M_{fy} = -M_{fm} \operatorname{sign} \omega_y.$$

Thus, the nonlinear system of stabilization consists of both linear and nonlinear elements.

The main feature of nonlinear systems should be consider the appearing possibility of limit cycles in them - periodic undamped oscillations. Thus, amplitude of those undamped oscillations is independent of external influence as well as of initial conditions. In general, the limit cycle may be not sinusoidal.

In this regard in researching of appearing possibility of limit cycles in nonlinear systems of stabilization, determination of their parameters and stability analysis, synthesis of corrective devices, which eliminating the appearance in nonlinear system limit cycle, present considerable interest.

3. Solution of the problem

Consider the stabilization system, angle sensor which has not error $\Delta\varphi$, caused by the action of perturbing moments M_x, M_y on the frame of three-stage gyro. If necessary, angle sensor, as a nonlinear element, can be investigated separately for checking the appearance of limit cycles in it.

The principle of superposition does not apply to nonlinear systems, so, we will consider the system that is only under the action of the control signal φ_c. Because of adopted limitations, the block diagram of a nonlinear stabilization system acquire a typical kind.

After the convolution of contour I and the transition to the frequency domain, we obtain an equivalent of frequency transfer function of the linear part of the stabilization system

$$W(j\omega) = \frac{k_G k_0 G_m}{\left[-(T_0 + T)\omega + j\left(1 + k_D k_0 D_m - T_0 T \omega^2\right)\right]\omega}. \tag{1}$$

We will apply to the nonlinear element method of harmonic linearization and will find in the handbook on automation the function which describe it [1,2]

$$W_f(a_m, \omega) = \frac{N_1 + jC_1}{a_m} = k - \frac{2k}{\pi}\left(\arcsin\frac{b}{a_m} + \frac{b}{a_m}\sqrt{1 - \frac{b^2}{a_m^2}}\right), \tag{2}$$

where b is width of the insensibility zone; k is the gain coefficient in the linear regime.

Note that the fundamental difference between harmonic linearization and usual linearization is that if we have harmonic linearization, the nonlinear characteristic we will change to linear, the slope of which depends on the amplitude of the signal at the input of the nonlinear element.

In a closed system can predict the existence of a limit cycle, if for some values of the amplitudes a_{m0} and frequencies ω_0 of the signal at the input of the nonlinear element the amplitude and phase of the frequency response characteristic (APFRC) of open circuit will be equal to

$$1+W(j\omega)W_f(a_m,\omega)=0. \tag{3}$$

Let rewrite equation (3) as

$$W(j\omega)=-\frac{1}{W_f(a_m,\omega)}. \tag{4}$$

The left-hand side of equation (4) is APFRC of linear component of stabilization system. The right-hand side - reverse APFRC of nonlinear element, taken with opposite sign. The parameters $W(j\omega)$ are determined only by the frequency of input signal and does not depend on its amplitude. But parameters like $W^{-1}(a_m,\omega)$, are on the contrary, determined only by the amplitude of the signal at the input of the nonlinear element and does not depend on its frequency.

Therefore, if it build graphical images on complex plane $W(j\omega)$ and $W_f^{-1}(a_m,\omega)$, then the point of its intersection will satisfy the expression (4). The obtained values of the amplitude a_{m0} and frequency ω_0 determine the changing law of the limit cycle.

When plotting the inverse APFRC of nonlinear element, we take into account the particular cases of calculation its describing function

$$W_f(a_m=b)=k-\frac{2k}{\pi}\left(\arcsin 1+1\sqrt{0}\right)=k-\frac{2k}{\pi}\cdot\frac{\pi}{2}=0;$$

$$W_f(a_m\to\infty)=k-\frac{2k}{\pi}\left(\arcsin 0+0\sqrt{1}\right)=k.$$

Graphical representation of $W^{-1}(a_m,\omega)$ in the complex plane (Figure 2) is a straight line coinciding with the negative direction of the x- axis. The maximum value equal to $-k^{-1}$ the inverse APFRC reaches with $a_m\to\infty$.

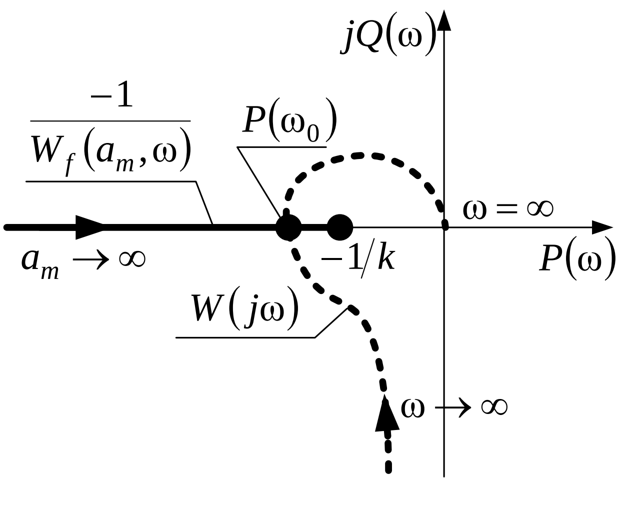

Figure 2. Parameters of the limit cycle.

On the basis of the frequency transfer function (1) of the linear component of the system, we find an algorithm calculating its APFRC

$$W(j\omega)=\frac{-k_G k_0 G_m (T_0+T)}{\left[(T_0+T)^2\omega^2+\left(1+k_D k_0 D_m-T_0 T\omega^2\right)^2\right]\omega}--j\frac{k_G k_0 G_m \left(1+k_D k_0 D_m-T_0 T\omega^2\right)}{\left[(T_0+T)^2\omega^2+\left(1+k_D k_0 D_m-T_0 T\omega^2\right)^2\right]\omega}. \quad (5)$$

APFRC graph $W(j\omega)$ on the complex plane, when the frequency ω signal is changes from 0 to ∞ is presented in Fig.2 by dotted line.

Let find the coordinate of the point of intersection $W(j\omega)$ and $W_f^{-1}(a_m,\omega)$. First of all we determine from (5) the frequency of possible limit cycle

$$\omega_0=\sqrt{\frac{1+k_D k_0 D_m}{T_0 T}}.$$

Substituting the value ω_0 into the real component of equation (5), we find

$$P(\omega_0)=\frac{-k_G k_0 G_m T_0 T}{(T_0+T)(1+k_D k_0 D_m)}.$$

A limit cycle will be possible if $\left|\frac{1}{k}\right|<P(\omega_0)$, so

$$k>\frac{(T_0+T)(1+k_D k_0 D_m)}{k_G k_0 G_m T_0 T}.$$

The amplitude of a possible limit cycle a_{m0} we find, in accordance with equation (4), after the substitution of values in it for $W(j\omega_0)$ and $W^{-1}{}_f(a_m,\omega)$

$$\frac{-k_G k_0 G_m T_0 T}{(T_0+T)(1+k_D k_0 D_m)}=\frac{-1}{k-\frac{2k}{\pi}\left(\arcsin\frac{b}{a_m}+\frac{b}{a_m}\sqrt{1-\frac{b^2}{a_m^2}}\right)}.$$

In practice, the amplitude is determined by the tables function $W_f(a_m,\omega)$ for found value of left-hand side of last equation if there is information about the parameter *b* of the nonlinear element.

According to the received data we write the variation law of possible limit cycle

$$\varphi_0(t)=-\varphi(t)=-a_{m0}\sin\omega_0 t.$$

In the stable limit cycle the oscillation amplitude returned to its previous value after its change, that is caused by this or any other perturbation. Otherwise, the limit cycle is unstable. If, for example, in a nonlinear system of stabilization an unstable limit cycle is possible, then with amplitude decreasing, due to any factor, these oscillations are damped with time. Conversely, if the oscillation amplitude increases, then it will increase without limit or in system a new limit cycle will arise with another amplitude or with another frequency.

Stability of limit cycle can be estimated, for example, using the criterion of Goldfarb [3]. In accordance with Fig.2 we make a conclusion about the unstable limit cycle in the considering nonlinear system.

In any case, it is desirable to avoid the occurrence of limit cycles. This issue can be resolved by introducing into the nonlinear system the correction circuits.

We denote the transfer function of the corrective circuit $W_{cc}(s)$. Applying to the corrected nonlinear system of stabilization method of describing functions we can write

$$W_{cc}(j\omega)W(j\omega) = -W_f^{-1}(a_m, \omega).$$

Thus, the frequency transfer function of the linear part of the corrected system is determined by the product of the frequency transfer functions of the corrective circuit and of linear component. Therefore, APFRC of the linear part of the corrected system will be a scaled APFRC of its linear component.

Corrective circuit should be chosen with such frequency transfer function $W_{cc}(j\omega)$, to prevent the crossing in the complex plane of inverse APFRC of nonlinear element and of APFRC of linear part of corrected system.

Let introduce into the nonlinear system of stabilization a corrective circuit with gain coefficient k_{cc}.

Limit cycle will be excluded if the coordinate of the point of intersection of the corrected APFRC of the linear part of the system of stabilization $k_{cc}P(\omega_0)$ will not exceed the maximum value of the inverse APFRC of nonlinear element $\left|k^{-1}\right|$

$$\left|\frac{1}{k}\right| > \frac{k_{cc}k_G k_0 G_m T_0 T}{(T_0 + T)(1 + k_D k_0 D_m)}.$$

From the last inequality we find the value of the gain coefficient of the corrective circuit

$$k_{cc} < \frac{(T_0 + T)(1 + k_D k_0 D_m)}{k k_G k_0 G_m T_0 T},$$

which ensures the exclusion of occurrence of the limit cycle.

Conclusions

The presence of nonlinearities in the stabilization system leads to the possibility of occurrence in the system periodic undamped oscillations - limit cycles.

The existence of a limit cycle and its parameters can be predicted based on the analysis of APFRC of linear component of the system and of inverse APFRC of nonlinear element.

The introduction into the nonlinear stabilization system the corrective circuits will eliminate the possibility of occurrence of limit cycle in the system.

References

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