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State Observer in Orientation System of Small Space Vehicle

The possibility of using system state observers in the orientation and stabilization systems of small space vehicles is considered. A comparative assessment of systems with a system state observer and classical execution is carried out.

Aerospace technologies has had an increasing impact in recent decades on the economic and social development of states and societies. The emergence of a new class of spacecrafts - small space vehicles (SSV), allows you to move from grand space projects to inexpensive - monitoring, information gathering, surveillance etc.

Promising direction of SSV development may be the creation of automated systems of orientation and stabilization (ASOS) with the use of system state observers.

In this connection the development of a system of orientation and stabilization of the SSV with a system state observer and its comparative assessment with the classical one is of some interest.

According to [1], we obtain the calculated model of the system of orientation and stabilization of the SSV (Fig. 1).

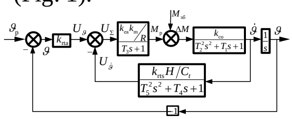


Fig. 1. Calculation model

In order to obtain more information about the studied system of angular orientation and stabilization of the SSV and more efficient design, the transition to its description in state variables was performed. The block diagram of the closed ASOS in state variables is presented in Fig. 2.

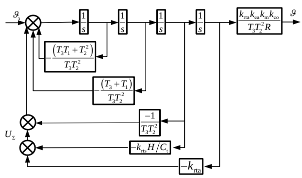


Fig. 2. Block diagram of a closed-loop system

The model (Fig. 2) makes it possible to find the matrices of coefficients $\mathbf{A}_f, \mathbf{B}$ input and $\mathbf{C}$ output of the linear matrix equation of the closed ASOS SSV

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_f \mathbf{x} + \mathbf{B}\vartheta_{\text{p}}. \\ \vartheta &= \mathbf{C}\mathbf{x}. \end{aligned} \qquad (1)$$

We find

$$\mathbf{A}_f = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_{\text{rta}} & \dfrac{-1}{T_3 T_2^2} - k_{\text{rts}} H / C_{\text{t}} & -\dfrac{(T_3 + T_1)}{T_3 T_2^2} & -\dfrac{(T_3 T_1 + T_2^2)}{T_3 T_2^2} \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} \dfrac{k_{\text{rta}} k_{\text{ea}} k_{\text{m}} k_{\text{co}}}{T_3 T_2^2 R} & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Let's now move on to ASOS with a system state observer.

Suppose we have an SSV controlled by the original coordinate. The equations of its motion have the form

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\vartheta_{\text{p}} \\ \vartheta &= \mathbf{C}\mathbf{x}. \end{aligned} \tag{2}$$

It is necessary to obtain an estimate of the SSV state vector $\mathbf{x}(t)$, which we denote as $\widehat{\mathbf{x}}(t)$. The functional diagram of the state vector estimation is shown in Fig. 3. In the evaluation process, all available information can be used, i.e., the input signal $\vartheta_{\text{p}}(t)$, the measured value of the output $\vartheta(t)$ and the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$.

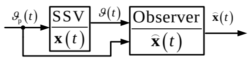


Fig. 3. Functional diagram for estimating the SSV state vector

Since the system state observer must have the same dynamics as the SSV, we will write its equation as

$$\dot{\widehat{\mathbf{x}}}(t) = \mathbf{F}\widehat{\mathbf{x}}(t) + \mathbf{H}\vartheta_{\text{p}}(t) + \mathbf{G}y(t)\ . \tag{3}$$

Matrices $\mathbf{F}, \mathbf{G}, \mathbf{H}$ should be chosen so as $\widehat{\mathbf{x}}(t)$ to give an accurate estimate $\mathbf{x}(t)$. Equations for determining matrices $\mathbf{F}, \mathbf{G}, \mathbf{H}$ can be obtained in different ways. Let's use the method of transfer function.

The Laplace transform of equations (2) gives

$$\begin{aligned} s\mathbf{X}(s) &= \mathbf{A}\mathbf{X}(s) + \mathbf{B}J_{\text{p}}(s) \\ J(s) &= \mathbf{C}\mathbf{X}(s). \end{aligned}$$

Solve these equations for $\mathbf{X}(s)$:

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}J_{\text{p}}(s)\,, \tag{4}$$

where $(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} = \dfrac{\mathbf{X}(s)}{J_{\text{p}}(s)}$ is the matrix transfer function.

Transforming the system state observer equation (3) according to Laplace, we find its matrix transfer function

$$\frac{\widehat{\mathbf{X}}(s)}{J_{\mathrm{p}}(s)} = (s\mathbf{I} - \mathbf{F})^{-1}\left[\mathbf{H} + \mathbf{GC}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\right]. \quad (5)$$

We equate the matrix transfer functions (4), (5)

$$(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} = (s\mathbf{I} - \mathbf{F})^{-1}\left[\mathbf{H} + \mathbf{GC}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\right]$$

and find the system state observer matrices

$$\mathbf{F} = \mathbf{A} - \mathbf{GC}; \quad \mathbf{H} = \mathbf{B}. \quad (6)$$

On the basis of (3) and (6) we find the equation of the system state observer

$$\dot{\widehat{\mathbf{x}}}(t) = (\mathbf{A} - \mathbf{GC})\widehat{\mathbf{x}}(t) + \mathbf{B}\vartheta_{\mathrm{p}}(t) + \mathbf{G}\vartheta(t),$$

where $\mathbf{G}$ matrix is to be determined.

When implementing the control law of the SSV, the observer is placed in the control circuit, and the signal received at the input of the automatic control system is a combination of all variables of the SSV state.

Thus, the task of observer synthesis is only to determine the matrix $\mathbf{G}$ by the desired characteristic polynomial $A_{\mathrm{od}}(s)$ of the observer and the known $\mathbf{A}$ and $\mathbf{B}$ matrices of SSV.

We find the characteristic equation of the system state observer $\det(s\mathbf{I} - \mathbf{A} + \mathbf{GC}) = 0$.

Since the speed of the observer should be in 2-4 times higher than the speed of the system [2] then we choose his desired characteristic equation

$$A_{\mathrm{od}}(s) = s^4 + a_{3\mathrm{o}}s^3 + a_{2\mathrm{o}}s^2 + a_{1\mathrm{o}}s + a_{0\mathrm{o}}.$$

Then the matrix $\mathbf{G}$ can be found from the equation

$$\det(s\mathbf{I} - \mathbf{A} + \mathbf{GC}) = A_{\mathrm{od}}(s).$$

Using the data of the parameters of small space vehicles and their control systems, after calculating all the necessary matrices we find the final equation (7) of the SSV system state observer.

$$\begin{bmatrix} \dot{\widehat{x}}_1 \\ \dot{\widehat{x}}_2 \\ \dot{\widehat{x}}_3 \\ \dot{\widehat{x}}_4 \end{bmatrix} = \begin{bmatrix} 0.14 & 1 & 0 & 0 \\ -11.832 & 0 & 1 & 0 \\ 72.656 & 0 & 0 & 1 \\ -469.007 & -26.67 & -12 & -7.87 \end{bmatrix} \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \\ \widehat{x}_3 \\ \widehat{x}_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \vartheta_{\mathrm{p}} + \begin{bmatrix} 0.006 \\ 0.569 \\ -3.493 \\ 21.548 \end{bmatrix} \vartheta. \quad (7)$$

The last algorithm allowed us to simulate (Fig. 4) the state observer and conduct research on its capabilities as part of the orientation and stabilization system of the small space vehicles.

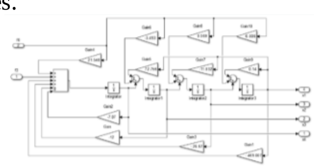


Fig. 4. System state observer model

Based on the models of an open-loop system and state observers, taking into account the formation of the control law $u(t) = -\mathbf{K}\hat{\mathbf{x}}(t)$ we develop a complete model of an automated system of orientation and stabilization of a small space vehicle with a system state observer. The model is shown in Fig. 5. Here, the models of the open-loop orientation and stabilization system, as well as the system state observer, are reduced to the level of subsystems.

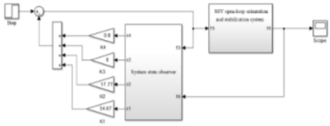


Fig. 5. ASOS SSV model with a system state observer

The developed models (Fig. 2, 5) made it possible to carry out a comparative assessment of the orientation and stabilization systems of a small space vehicle with a system state observer and in the classical version.

Fig. 6 shows the fields of zeros and poles of the systems under study. The upper field corresponds to the ASOS with a system state observer, the lower one - to the classical one.

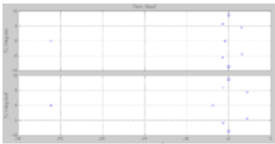


Fig. 6. Fields of zeros and poles

The analysis of the given material shows that the margin of stability of ASOS with the system state observer is 9% higher, than in classical. The dynamics of the systems are similar.

Since the tracking mode for ASOS is the main one when adjusting the angular orientation of the SSV according to the signals of the Flight Control Center, the estimation of tracking errors $\theta_{tr}(t)$ of the studied systems was performed. The laws of change in time of the control signal $\vartheta_{p}(t)$ and the output signals of the ASOS with the system state observer $\vartheta_{sso}(t)$ and the classical $\vartheta_{cl}(t)$ are given in Fig. 7.

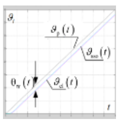
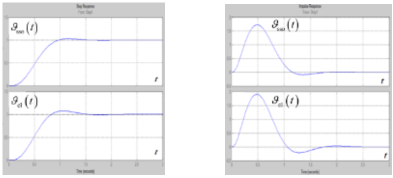


Fig. 7. Characteristics in the tracking mode

The discrepancy of tracking errors for the considered systems does not exceed 0,2%.

The dynamics of the systems is illustrated by the transient and impulse transient characteristics obtained during the experiment. The characteristics are shown in Fig. 8. They confirm the preliminary conclusion about the similarity of the dynamic processes of ASOS.

It has been established that the control signal response time in the ASOS SSV with the system state observer is slightly higher than the analogous indicator of the classical ASOS. On the other hand, the regulation time of the ASOS of the small space vehicle with the system state observer is less than in the classical one. The over-regulation in the ASOS with the system state observer is 5%, in the classical – 12%.



a b

Figure 8. Dynamic characteristics of ASOS SSV:
a-transient; b-pulse transients

Since the authors did not have accurate information about random perturbations, an assumption was made about their unlimited power. In the theory of automatic control, the role of a random process with infinitely high energy is played by the so-called white noise.

The spectral densities of the SSV ASOS reactions to white noise are shown in Fig. 9.

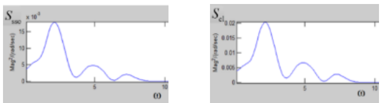


Fig. 9. Spectral densities of output signals

It was found that the frequency spectra of the ASOS with the систем state observer and the classical one coincide. At the same time, the root-mean-square value of the error of the system with an observer is only 0.8% higher than that of the classical ASOS.

Conclusions. A promising direction in the development of SSV can be the creation of automated orientation and stabilization systems with the use of system state observers.

The developed mathematical models allowed to carry out a comparative assessment of the system of classical construction with the system, which includes a system state observer.

According to the research results, it has been established that the use of a system state observer in the orientation and stabilization systems of small space vehicles will allow not only to reduce the weight and overall dimensions of the small space vehicle, increase their reliability and reduce the cost of components, but also provide practically the same control quality indicators as in the classical version ASOS.

References

1. Ablesimov A.K. et al. Automatic Control of Moving Objects. - Kharkov: NTU “KhPI”, 2008. – 443 p.
2. Ablesimov A.K. The theory of automatic control. - K: Osvita Ukrainy, 2019. – 269 p.
3. Sokolov Y.N. Computer analysis and design of control systems. - Kharkov: NTU “KhAI”, 2010. – 343 p.