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### **Fuzzy functions of determining the dependence of potential risk/loss value on the time of flight completion in flight emergency**

*Based on the mathematical apparatus of fuzzy sets theory, the dependence between the time of flight completion and the value of potential risk/loss in flight emergency was determined. The obtained dependencies can be used in the Decision Support System to predict the optimal time of performing collaborative technological procedures for flight emergency parring by the Air Navigation System's operators.*

**Problem statement.** In the process of collaborative decision making (CDM) by the operators of the Air Navigation System (ANS) in flight emergency (FE), the problem of choosing the optimal alternative for flight completion occurs. To solve it in certainty conditions are used: in certainty conditions – the network models based on the time assessment of technological procedures  $T$  for FE parring by the ANS's operators, in stochastic uncertainty (risk) conditions – a decision tree based on the assessing potential risk  $R$  is used, in non-stochastic uncertainty conditions – the decision matrix based on the assessment of potential loss  $Q$  [1]. In this case, the potential risk  $R$  or potential loss  $Q$  is nonlinearly dependent on the time, that is, minimizing the time of flight completion in FE  $T$  does not always lead to a minimization of potential risk/loss  $R/Q$ , but may on the contrary provoke dangerous consequences due to lack of time on parry of the situation and increasing the probability of FE developing toward the deterioration.

**The purpose of the publication** is determine the dependence between the value of potential risk/loss in FE and the time of flight completion based on the mathematical apparatus of fuzzy sets theory.

**Main part.** Effectiveness criteria of CDM by the ANS's operators FE are optimizing the time of flight completion  $T$  (under certainty) (1), minimizing potential risk  $R$  (under stochastic uncertainty) (2), or minimizing potential loss  $Q$  (under non-stochastic uncertainty) (3) as a result of the flight completion:

$$A_{opt}^{CDM} = opt T(t); \quad (1)$$

$$A_{opt}^{CDM} = min R(T, \theta, P, Q); \quad (2)$$

$$A_{opt}^{CDM} = min Q(T, \theta, Y), \quad (3)$$

where  $T$  – is the time of flight completion;

$t$  – is the time of performing technological procedures for FE parring by the ANS's operators;

$\theta = \{\theta_{jj}\}$  – is the set of factors that affect the results of CDM by the ANS's operators;

$P = \{p_{jj}\}$  – is the set of objective and subjective assessments of the probability of the factors' influence;

$Q = \{q_{jj}\}$  – is the set of results of choosing alternative collaborative decisions (the results of the consequences  $y \in Y$ );

$Y = \{y_{jj}\}$  – is the set of consequences of choosing alternative collaborative decisions.

Nonlinearity of dependence between potential risk/loss  $R/Q$  and the time of flight completion  $T$  in FE can be represented in the form of Fig. 1.

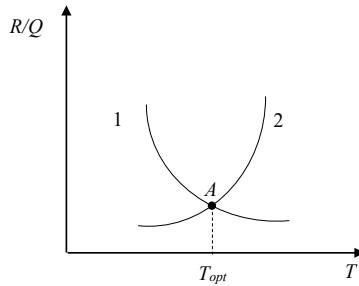


Fig. 1. The dependence between potential risk/loss  $R/Q$  and the time of flight completion  $T$  in FE

From the graph of the function  $T = f(R/Q)$  in Fig. 1, it can be seen that when the time of flight completion  $T$  increases then the value of potential risk/loss  $R/Q$  can both decrease due to increasing the probability of successful parry of FE by the ANS's operators through the time reserve (graph 1), and increase due to increasing the probability of FE developing towards deterioration (graph 2). Optimal time of the flight completion  $T_{opt}$  is at the point of intersection of hyperboles 1 and 2 – point A.

To obtain the value of the potential risk/loss  $R/Q$ , which depends on the time of flight completion  $T$ , was used mathematical apparatus of fuzzy set theory [2].

A fuzzy number  $A$  is presented in an  $LR$ -form that corresponds to the LEFT and RIGHT parts of the functions [2] (4):

$$\mu^A(x = P(R/Q)) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), \alpha > 0, \forall x \leq m; \\ R\left(\frac{x-m}{\beta}\right), \beta > 0, \forall x \geq m, \end{cases} \quad (4)$$

where  $m$  – is the average value of a fuzzy number  $A$ ;

$\alpha$  – deviation to the left;

$\beta$  – deviation to the right.

If  $\alpha = \beta = 0$ , then a fuzzy number  $A$  goes into a clear number  $m$ .

So, the  $LR$ -form of a fuzzy number  $A$  can be represented as a trinity  $A = (mA, \alpha A, \beta A)$ . Functions  $L$  and  $R$  have the following properties:

- parity:  $L/R(-x) = L/R(x)$ ;
- normalization:  $L/R(0) = 1$ .

In practice,  $LR$ -representation is simplified by using linear functions, leading to triangular fuzzy numbers that have the membership function like (5) [2]:

$$\mu^A(x = R/Q) = \begin{cases} 0, & \text{if } a^+ \leq x \leq a^-; \\ \frac{x-a^-}{a-a^-}, & \text{if } a^- \leq x \leq a; \\ \frac{a^+-x}{a^+-a}, & \text{if } a \leq x \leq a^+, \end{cases} \quad (5)$$

where  $a^-$  – is the lower modal value of a fuzzy number  $a$ ;  
 $a^+$  – is the upper modal value of a fuzzy number  $a$ .

The statistical method of processing expert information was used [2], in which as the degree of belonging of the element  $a \in A$  to the set  $X$  is accepted the estimation of the frequency of use by an expert of a fuzzy concept to characterize an element.

The anonymous individual survey was attended by 30 respondents (pilots, air traffic controllers, UAV operators, flight dispatchers, service agents, maintenance engineers, etc.) by the Delphi method [3]. Experts were asked to evaluate the impact of each qualitative value of the time of flight completion  $T_j$  on a qualitative value of potential risk/loss  $R_i/Q_i$ , which it causes, as follows (6):

$$R_i/Q_i = \begin{cases} 1, & \text{If the expert thinks that } R_i/Q_i \text{ is a consequence of } T_j; \\ 0, & \text{If the expert thinks that } R_i/Q_i \text{ isn't a consequence of } T_j. \end{cases} \quad (6)$$

The value of potential risk/loss  $R_i/Q_i$  can take the following values on a quality scale that is formulated in terms of fuzzy setting theory: a very low risk/loss  $R_1/Q_1$  – corresponds to economic losses (0-0,20); a low risk/loss  $R_2/Q_2$  – corresponds to damage of aviation technic (0,21-0,40); an average risk/loss  $R_3/Q_3$  – corresponds to the incident (0,41-0,60); a high risk/loss  $R_4/Q_4$  – corresponds to the accident (0,61-0,80); a very high risk/loss  $R_5/Q_5$  – corresponds to the catastrophe (0,81-1,00).

The time of flight completion  $T_j$  can take the following values on a quality scale that is formulated in terms of fuzzy setting theory: an insignificant time  $T_1$  – corresponds to the time of transition of the flight situation to complicated; a moderate time  $T_2$  – corresponds to the time of transition of the flight situation to complex; a considerable time  $T_3$  – corresponds to the time of transition of the flight situation to emergency; a maximum permissible time  $T_4$  – corresponds to the time of transition of the flight situation to catastrophic.

The maximum possible value of the time of flight completion  $T$  was conventionally accepted 1. Interval  $[0; 1]$  was divided into 4 segments, each of which calculated the number of experts who believe that the value of potential risk/loss  $R_i/Q_i$  is a consequence of the time of flight completion  $T_j$ . The accuracy of expert assessments proves the obtained value of the coefficient of variation that does not exceed 33%  $v_{av} = 28\%$ .

The results of the evaluation of the terms of the linguistic variable "The value of potential risk/loss" are given in Table 1.

Table 1.

The results of the evaluation of the terms of the linguistic variable "The value of potential risk/loss"

$\mu(R_i/Q_i)$	The time of flight completion, $T_j$			
	Insignificant time, $T_1$	Moderate time, $T_2$	Considerable time, $T_3$	Maximum permissible time, $T_4$
$\mu(R_1/Q_1)$	0,00	1,00	0,82	0,00
$\mu(R_2/Q_2)$	0,28	1,00	0,67	0,19
$\mu(R_3/Q_3)$	0,84	0,97	1,00	0,73
$\mu(R_4/Q_4)$	0,88	0,17	0,47	1,00
$\mu(R_5/Q_5)$	0,95	0,05	0,20	0,98

The membership functions for each linguistic variable "The value of potential risk/loss" are presented in Fig. 2.

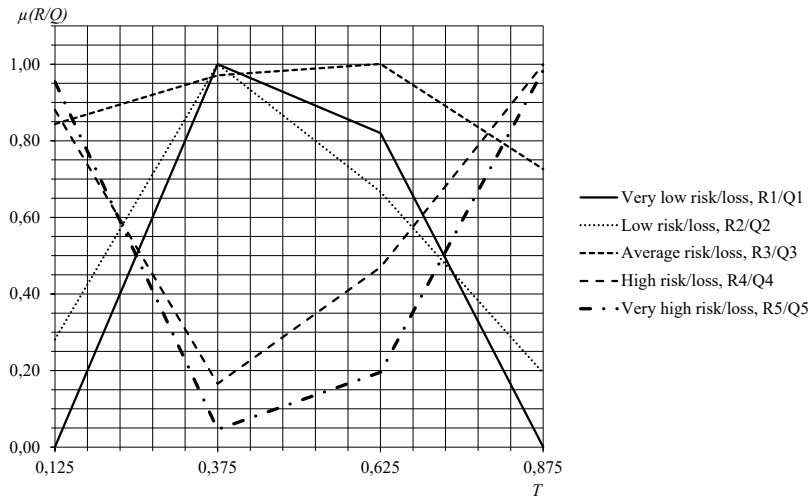


Fig. 2. The membership functions of term-set "The value of potential risk/loss"

From Fig. 2 is determined that the insignificant time of flight completion  $T_1$  in FE leads to very high risk/loss  $P_5(R/Q)$ ; moderate time  $T_2$  – to very low  $P_1(R/Q)$ , low  $P_2(R/Q)$  or average  $P_3(R/Q)$  risk/loss; considerable time  $T_3$  – to average risk/loss  $P_3(R/Q)$ ; maximum permissible time  $T_4$  – to high  $P_4(R/Q)$  or very high  $P_5(R/Q)$  risk/loss.

**Conclusion.** Based on the fuzzy sets theory the dependence between the time of flight completion and the value of potential risk/loss in FE was determined. The obtained dependencies can be used in the Decision Support System to predict the optimal time of performing collaborative technological procedures for FE parring by the ANS's operators.

## References

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