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Aircraft control optimization during landing approach according to the minimax criterion

The article solves the problem of optimization of aircraft control during landing approach according to the minimax criterion. The case of lateral movement is investigated to restore the position along the runway axis in the minimum time under extreme external disturbances, including the maximum allowable crosswind. Minimization of the time functional is carried out due to the extreme control limited by the control rudder deflection angles.

Landing is the final difficult and least reliable stage of the flight, where all external factors that complicate the crew's activities are most affected. It is carried out by almost one person - the pilot, whose activity changes dramatically from monotony to maximum mobilization of attention and professional skills, from instrument flight to visual flight. Correction of landing errors as you approach the ground is limited by extremely small reserves of time and space. The aircraft on landing has deteriorated characteristics of stability and controllability.

The complication of flight activity, especially at the landing stage, necessitates detailed control and analysis of the quality of control at this stage.

When controlling an aircraft on landing, the tasks of guaranteed optimal control in terms of speed with known restrictions on disturbing influences and the construction of admissible initial states of the object, for which the landing problem is successfully solved, that is, it is necessary to construct the area of permissible deviations at the decision height, from any point which the aircraft can be brought out with manual control to a given area of runway [2].

The greatest difficulties are caused by the compensation of lateral deviations z and $\dot{z}$ therefore we will consider only the lateral movement. The position of the aircraft is determined by the coordinates of the center of mass, yaw angle ψ, bank angle γ, slip angle $\beta = \psi + (\dot{z} - W_z)/V$ (Fig. 1.).

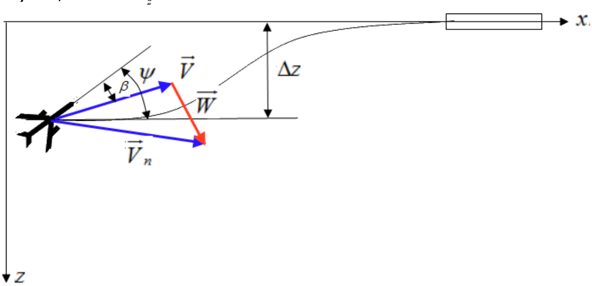

Fig. 1.

We consider the equations of lateral motion in the form

$$
\begin{aligned}
\ddot{z} &= a_{z\beta}\beta + a_{z\gamma}\gamma + a_{z\delta_r}\delta_r \\
\ddot{\psi} &= a_{\psi\beta}\beta + a_{\dot{\psi}}\dot{\psi} + a_{\psi\dot{\gamma}}\dot{\gamma} + a_{\psi\delta_r}\delta_r + a_{\psi\delta_a}\delta_a \\
\ddot{\gamma} &= a_{\gamma\beta}\beta + a_{\gamma\dot{\psi}}\dot{\psi} + a_{\dot{\gamma}}\dot{\gamma} + a_{\dot{\gamma}\delta_r}\delta_r + a_{\dot{\gamma}\delta_a}\delta_a
\end{aligned} \qquad (1)
$$

δ_r - angle of deflection of the rudder;

δ_a - aileron deflection angle;

The coefficients a_{ij} are determined by the mass and aerodynamic characteristics of the aircraft.

$$
\begin{array}{llll}
a_{z\beta} = \dfrac{qSc_z^{\beta}}{m}; & a_{\psi\delta_r} = a_{\psi\beta}\dfrac{m_y^{\delta_r}}{m_y^{\beta}}; & a_{\dot{\psi}} = \dfrac{qSl^2}{2I_yV}; & a_{\dot{\gamma}\delta_r} = a_{\gamma\beta}\dfrac{m_x^{r}}{m_x^{\beta}}; \quad (2) \\
a_{z\gamma} = g; & a_{\psi\delta_a} = a_{\psi\beta}\dfrac{m_y^{\delta_a}}{m_y^{\beta}}; & a_{\psi\dot{\gamma}} = a_{\dot{\psi}}\dfrac{m_y^{\overline{\omega_x}}}{m_y^{\overline{\omega_y}}}; & a_{\dot{\gamma}\delta_a} = a_{\gamma\beta}\dfrac{m_x^{a}}{m_x^{\beta}}; \\
a_{z\delta_r} = \dfrac{a_{z\beta}c_z^{r}}{c_z^{\beta}}; & a_{\gamma\beta} = a_{\psi\beta}\dfrac{I_y m_x^{\beta}}{I_x m_y^{\beta}}; & a_{\gamma\dot{\psi}} = a_{\dot{\psi}}\dfrac{I_y m_x^{\overline{\omega_y}}}{I_x m_y^{\overline{\omega_y}}}; & \\
a_{\psi\beta} = \dfrac{qSc_z^{\beta}}{m}; & a_{\dot{\gamma}} = a_{\gamma\dot{\psi}}\dfrac{m_x^{\overline{\omega_x}}}{m_x^{\overline{\omega_y}}}; & q = \dfrac{\rho V^2}{2} &
\end{array}
$$

Here S - wing area, l - wing span, m - aircraft mass, g - free fall acceleration, I_i - moments of inertia about the corresponding axes, q - velocity head, ρ - air density, c_i ,m_i - dimensionless aerodynamic coefficients, c_i^j, m_i^j - derivatives of the coefficients with respect to the corresponding variables, V-airspeed.

But on the pre-landing straight, the value of V is stabilized by changing the thrust of the engines. Therefore, in what follows, the value of V, and hence the coefficients a_{ij}, will be considered constant. Thus, the object is described by a system of linear equations with constant coefficients of the sixth order.

Thus, we have obtained a complex system of equations, which can be simplified by separating the program movement, disturbances and control errors. An important issue is to take into account the inertia of the control system links, such as the pilot and the aircraft control system, but we will consider slowly changing variables, these include the position and speed of the aircraft's center of mass.

Then the trajectory of the lateral movement is determined by the first equation, the weighting factor $a_{z\gamma}$ and the roll control.

Optimization of control during such a movement consists in the speed of reaching a given point of the trajectory at the point of the beginning of the alignment. From a mathematical point of view, in this case, we consider the problem of speed using the Pontryagin maximum principle [1].

In this case, we consider a second-order system

$$z = f(\dot{\gamma}, \gamma) \qquad (3)$$

$$\dot{z} = f(\dot{\gamma}, \gamma) \qquad (4)$$

The control action

$$u = g\sin\gamma \qquad (5)$$

is limited by the roll angle $|\gamma| \le \gamma_0$.

In the landing approach conditions, the initial conditions for lateral deflection and drift of the aircraft z_0 and $\dot{z}_0$. The problem is to choose such a control (5) that satisfies condition (6), under which the system passes from the initial state to the origin of coordinates, that is, to the position $z(T)$ and $\dot{z}\ (T)$ in the minimum time. Here T - the time of the beginning of the alignment, which should be chosen as minimum. This problem is equivalent to the requirement to minimize the functional

$$J = \int_0^T dt \qquad (6).$$

Hamilton function $H = p_1\dot{z} + p_2 u + 1$,

where p_1 and p_2 auxiliary variables satisfy the equations of maximum H . Thus, the optimal control is the relay control, which takes the values $u(t) = u_0 sign$ $p_2(t)$, and the change of sign occurs at the time moments at which the function $p_2(t)$ crosses the zero level. The quality of control in this case is the minimization of the maneuver time and the preparedness of the pilot to ensure the implementation of optimal control during the landing approach.

When landing, there are segments of automatic (semi-automatic) control from the point of entry into the glide path to the decision height and a segment of manual control from the decision height to the start point of alignment, where $z(T) = 0$ and $\dot{z}(T) = 0$.

Lateral motion modeling was carried out with the support of the Simulink software package [3] for an aircraft with a mass m=10^5kg and an approach speed V=250 km/h.

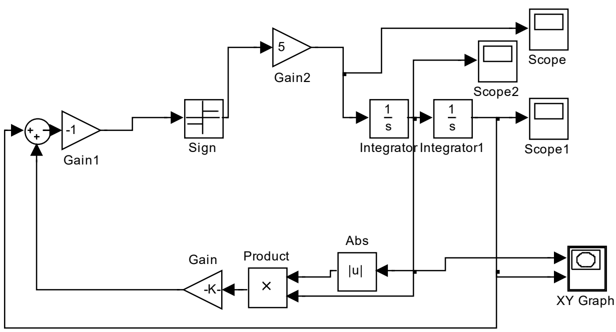

Fig. 2.

The general scheme of the simulation system is shown in Fig.2. The parameters of all links were adjusted taking into account the characteristics of the aircraft; the roll angle γ was taken as the control signal u. The initial parameters in the simulation were the lateral deviation Δz, and also $\dot{z}$, which implies the initial drift velocity or the presence of a heading error. Figure 3 shows the trajectory of lateral movement with roll $\gamma = 30^0$, initial lateral deviation $\Delta z = 20^0$ m and side wind W=0 m/s and W=15 m/s. The time to reach the leveling start point was equal to *t=11 s* in the absence of side wind w=0 m/s and lateral deviation $\Delta z = 200$ m and t=16 s with side wind w=0 m/s. In this case, attention should be paid to a simple graph of the trajectory of movement in the absence of wind, from which it can be seen that the rudder shift is carried out symmetrically at half the temporary position and with a lateral deviation $\Delta z = 100$m. In the presence of a crosswind, the time and place of rudder shifting changes, which greatly complicates the pilot's task.

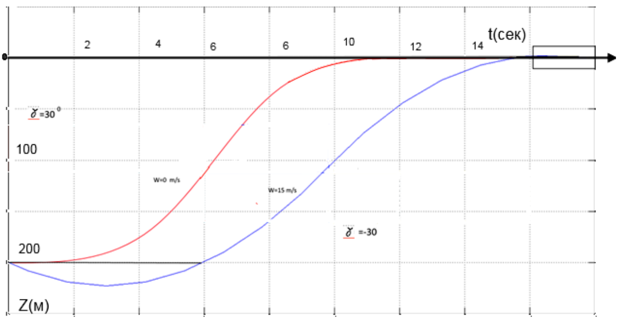

Fig. 3.

Figure 4 shows the lateral motion trajectories with roll $\gamma = 6^0$ for various crosswinds. Reducing the control roll improves flight safety, however, the recovery time and getting on the runway axis increases significantly and can reach one minute.

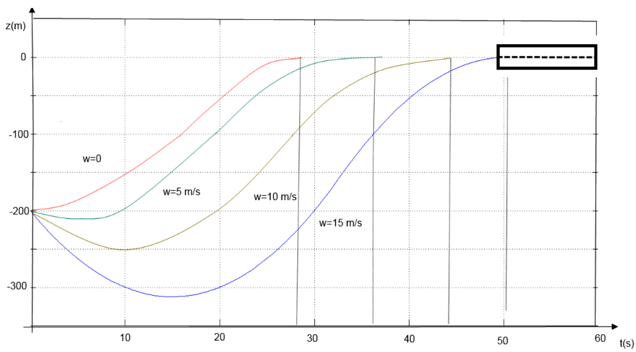

Fig. 4.

Figure 5 shows the phase trajectories of the optimal control of lateral motion. For solving the problems of synthesis of optimal control systems, the phase trajectories of the system, which reflect the deviation from the zero point with coordinates $\Delta z=$ 0m and $\dot{z}$ =0 m/sec, are important for representation. It can be seen from the graph that in order to parry initial deviations and accurately reach the end point, an accurate determination of the time and place on the trajectory for switching control is required. Phase coordinates, in addition to the theoretical value, are of great practical importance for the pilot, since they characterize the deviation from the runway axis and the speed of departure from the line of the given path and are reflected on the director instruments. The value of these parameters at the decision height can guarantee landing and exit to the leveling start point with zero values of these parameters.

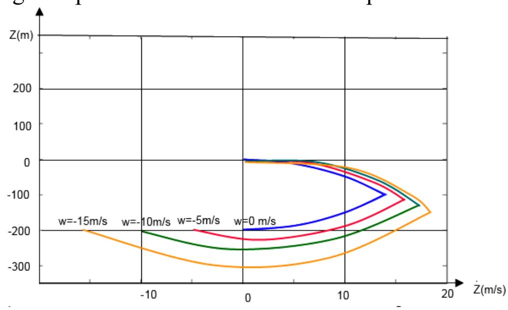

Fig. 5.

As an example, Figure 6 shows the permissible deviations of the parameters (phase coordinates) of the flight of the decision height for a guaranteed landing. The graph is built for control signals corresponding to the step deviation of the ailerons, providing roll $\gamma = 6^0$ and the estimated maneuver time *t=15 sec*. It should be noted that, in general, the graph is symmetrical in nature, corresponding to various deviations from the runway axis, wind direction and drift speed.

It can be determined from the graph that the maximum deviation, even in the absence of wind, is no more than $\Delta z= 50$ m, and when entering the runway axis with side drift at a speed of $\dot{z}$ = 6 m/s, optimal control can provide a guaranteed landing only after t=15 s. This time is theoretically minimal and serves to assess the quality of piloting when training cadets. On the practical side, to calculate the real time, it is necessary to take into account the inertia of the control links, as well as the human factor, which consists in assessing the professional training of the pilot. These factors can be taken into account by introducing control links in the form of transfer functions corresponding to the characteristics of the aircraft and the pilot. The main parameters characterizing the pilot (control link) are the time constant and the gain, which can be adjusted by professional training and evaluated in terms of compliance with the maximum theoretical values obtained as a result of mathematical modeling of the optimal aircraft control processes at such an important stage as landing approach and landing. These parameters, taking into account human factors, should provide a guaranteed landing within the specified deviation limits.

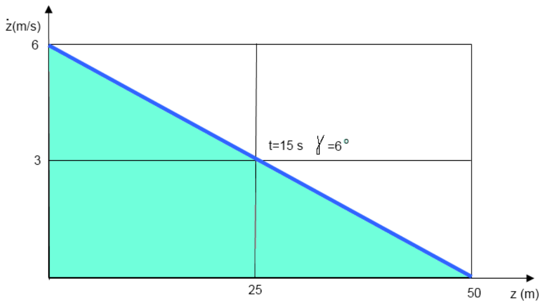

Fig. 6.

Conclusions

Using the methods of mathematical modeling, the following are determined in the work:

-optimal speed control parameters for aircraft at the stage of pre-landing maneuvering;

- limit deviations of phase coordinates to ensure safe landing;
- the results obtained can be used to assess the quality of management with different levels of professional training.

References

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