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Software-numerical optimization of coefficients of the Miller algorithm for a four-frequency model of angular motion of a rigid body

Refined coefficients of the Miller orientation algorithm, optimized for the analytical four-frequency model of the angular motion of a rigid body, were obtained. It is shown that the Miller algorithm with a new set of coefficients provides a smaller calculation error of the accumulated drift compared to the classic Miller algorithm and Ignagni's modification, which are optimized for conical motion.

Four-frequency analytical model of angular motion of a rigid body.

The mathematical model of the kinematics of the angular motion of an object as a rigid body is based on a sequence of four linear rotations, where the first three rotations are performed corresponding to the Krylov angles $\varphi(t)=k_1 t$, $\psi(t)=k_2 t$ and $\vartheta(t)=k_3 t$, and the fourth rotation is performed around the second axis rotated by the angle $\chi(t)=k_4 t$. The resulting quaternion $\Lambda(t)=(\lambda_0(t),\lambda_1(t),\lambda_2(t),\lambda_3(t))$ in this case will have the form:

$$\Lambda(t)=(\cos\frac{k_1 t}{2}+\vec{i}_3\sin\frac{k_1 t}{2})\circ(\cos\frac{k_2 t}{2}+\vec{i}_2\sin\frac{k_2 t}{2})\circ(\cos\frac{k_3 t}{2}+\vec{i}_1\sin\frac{k_3 t}{2})\circ(\cos\frac{k_4 t}{2}+\vec{i}_1\sin\frac{k_4 t}{2}),$$

where $\vec{i}_1,\vec{i}_2,\vec{i}_3$ are the orthos of the respective axes, and k_1,k_2,k_3 are the frequencies.

Projections $\omega_i(t)$ of the angular velocity vector of the body $\vec{\omega}(t)$ onto the connected axes can be obtained from the inverse kinematic equation $\vec{\omega}(t)=2\tilde{\Lambda}(t)\circ\frac{d}{dt}\Lambda(t)$, $\tilde{\Lambda}(t)$ is the conjugate quaternion to $\Lambda(t)$:

$$\omega_1(t)=-\sin(k_4 t)(k_1\cos(k_2 t)\cos(k_3 t)-k_2\sin(k_3 t)+\cos(k_4 t)(k_3-k_1\sin(k_2 t));$$

$$\omega_2(t)=k_4+\frac{1}{2}k_1(\sin((k_3+k_2)t)+\sin((k_3-k_2)t))+k_2\cos(k_3 t);$$

$$\omega_3(t)=\sin(k_4 t)(k_3-k_1\sin(k_2 t))+\cos(k_4 t)(k_1\cos(k_2 t)\cos(k_3 t)-k_2\sin(k_3 t)).$$

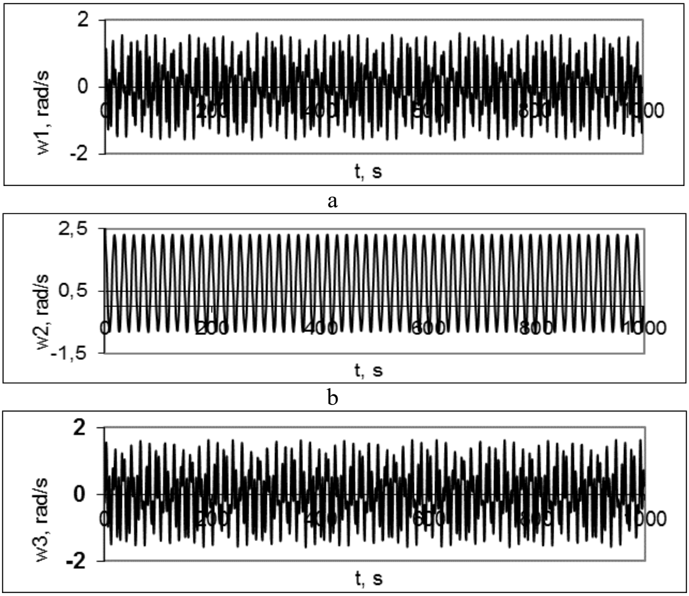
To model ideal signals at the output of angular velocity sensors in the form of quasi-coordinates, the components of the apparent rotation vector must first be found analytically $\vec{\theta}(t)=(\theta_1(t),\theta_2(t),\theta_3(t))=\int_0^t\vec{\omega}(t)dt,\ i=1,2,3$, and then use the formula

$$\theta^*_{ni}=\theta_i(t_n)-\theta_i(t_{n-1}),\ i=1,2,3.$$

The described kinematic model of angular motion is analytical, there are no errors associated with numerical integration in the quasi-coordinate values. Thus, it can be considered that a test movement has been built for evaluating the accuracy of orientation algorithms in SINS.

Software-numerical implementation of the angular motion model.

Figure 1 shows the time dependences of the projections of the angular velocity vector for the four-frequency model on the time interval $t \in [0,1000]$ s for the frequency values of the kinematic model $k_1 = 0.15$, $k_2 = 1.55$, $k_3 = 0.35$, $k_4 = 0.75$.



c

Fig. 1. Projections of the angular velocity vector of a rigid body

a – on the first axis; b – on the second axis; c – on the third axis

Software-numerical optimization of the Miller algorithm.

In the Miller algorithm [1], the increment of the orientation vector per calculation cycle $[t_{n-1}, t_n]$ is calculated by the formula:

$$\vec{\theta}_n = \vec{\theta}_n^* + \alpha(\vec{\theta}_n^1 \times \vec{\theta}_n^3) + \beta \vec{\theta}_n^2 \times (\vec{\theta}_n^3 - \vec{\theta}_n^1), \tag{1}$$

where $\vec{\theta}_n^* = (\theta_{n1}^*, \theta_{n2}^*, \theta_{n3}^*)$, quasi-coordinates $\vec{\theta}_n^1 = \int\limits_{t_{n-1}}^{t_{n-1}+1/3\Delta T} \vec{\omega}(t)dt$, $\vec{\theta}_n^2 = \int\limits_{t_{n-1+1/3\Delta T}}^{t_{n-1}+2/3\Delta T} \vec{\omega}(t)dt$, $\vec{\theta}_n^3 = \int\limits_{t_{n-1}+2/3\Delta T}^{t_{n-1}+\Delta T} \vec{\omega}(t)dt$ are formed within the calculation cycle at the points of removal of primary information $t_{n-1} + 1/3\Delta T$, $t_{n-1} + 2/3\Delta T$, $t_{n-1} + \Delta T$, ΔT – calculation cycle time. Miller obtained that $\alpha = 33/80$, $\beta = 57/80$. In the optimized Ignagni

[2] for the conical movement of the Miller algorithm, the values of the coefficients were specified: $\alpha = 36/80$, $\beta = 54/80$. The basis of the optimization will be the estimate of the error of the accumulated computational drift:

$$\delta\theta_n = 2arctg(|vect(\delta\Lambda_n|)/sqal(\delta\Lambda_n)),$$

where $\delta\Lambda_n$ is the quaternion of the accumulated orientation error $\delta\Lambda_n = \Lambda_n^* \circ \widetilde{\Lambda}_n$, $\Lambda_n^* = \Lambda^*(t_n)$ is the quaternion calculated by the orientation algorithm at the moment t_n, $\widetilde{\Lambda}_n$ – is the quaternion conjugated to the orientation quaternion of the four-frequency model. To obtain the calculated quaternion of the orientation Λ_n^*, we use the formula for the addition of turns $\Lambda_n^* = \Lambda_{n-1}^* \circ \Delta\Lambda_n^*$, where the quaternion $\Delta\Lambda_n^*$ calculated by the algorithm in the cycle of calculations $[t_{n-1}, t_n]$:

$$\Delta\lambda_n^* = 1-(1/8)\theta_n^2 + (1/384)\theta_n^4, \ \Delta\lambda_i^* = (1/2)\theta_{ni}(1-\theta_n^2/24), \ i = 1,2,3,$$

where θ_{ni} are the components of the orientation vector, $\theta_n^2 = \theta_{n1}^2 + \theta_{n2}^2 + \theta_{n3}^2$.

Modeling the test movement and evaluating the Miller algorithm will be carried out according to the block diagram shown in Fig. 2.

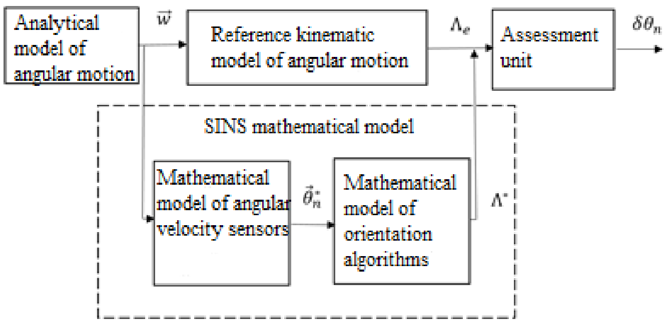


Fig. 2 – block diagram of modeling

The first stage of optimization. In Miller's classical algorithm (1) and in Ignagni's modification, the sum of the coefficients is $\alpha + \beta = 1.125$. Let's analyze the accuracy of Miller's algorithm and Ignagni's modification on the proposed four-frequency model, changing the sum of the coefficients $\alpha + \beta$ in the range from 1.125 to 1.128. Let's set the calculation cycle $\Delta T = 0.1$ s. The results of the numerical experiment are presented in Table 1. It was found that the minimum estimate of the drift error is observed not for $\alpha + \beta = 1.125$, as is the case for the classic Miller algorithm and Ignagni's modification, but for $\alpha + \beta = 1.127$.

The second stage of optimization. Let's now fix the sum of the coefficients in the algorithm $\alpha + \beta = 1.127$ and specify the coefficients α and β. To do this, we will change the coefficient α in the range from 0.8 to 1.2. The results of calculations of the error estimate of the accumulated drift by the modified Miller algorithm are presented in Table 2.

Table 1

The error of the Miller algorithm on the four-frequency model

$\alpha + \beta$	Miller's algorithm, drift (rad)	Ignagni modification, drift (rad)
1.1250	0.000556	0.000561
1.1260	0.000239	0.000244
1.1265	8.14E-05	8.59E-05
1.1270	7.84E-05	7.39E-05
1.1275	0.000236	0.000232
1.1280	0.000395	0.00039

Table 2

Accumulated computational drift error

α	Modified Miller algorithm, drift (rad)
0.80	3.21E-05
0.85	2.64E-05
0.90	2.09E-05
1.00	1.21E-05
1.04	1.06E-05
1.05	1.06E-05
1.06	1.07E-05

As a result of numerical optimization, it was found that the minimum error value of the accumulated computational drift occurs at the values of the coefficients in the Miller algorithm $\alpha = 1.05$, $\beta = 0.077$. At the same time, this error (1.06E-05 rad) is significantly smaller than the corresponding errors of the classic Miller algorithm (7.84E-05 rad) and the Ignagni modification (7.39E-05 rad).

Conclusions

A new analytical test motion of a rigid body based on a four-frequency kinematic model is presented. Software-numerical optimization of the coefficients of the Miller algorithm on this test motion is carried out, which ensures a minimum error of the accumulated computational drift.

References

1. Miller R.B. A new strapdown attitude algorithm//Journal of Guidance, Control and Dynamics. – Vol. 6. – No 4. – 1983. – P.287– 291.
2. Ignagni M.B. Optimal strapdown attitude integration algorithm//Journal of Guidance, Control and Dynamics. – Vol. 13. – No 2. – 1990. – P.363– 369.