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## Determining optimal aircraft maintenance task interval using average operational cost as a measure of efficiency

*Existing literature on optimal maintenance task models use the maintenance cost rate as an optimization criterion but overlook the reliability performance. Reducing the system maintenance cost rate does not imply that the system reliability performance is optimized in terms of cost, specifically for multicomponent systems which is the case for aircrafts. In this study, mathematical models for optimizing aircraft maintenance task intervals are developed and analyzed. These models quantify the preventive and corrective maintenance costs alongside the benefits of maintenance to obtain an optimum balance between both. The exponential and Erlang models of times between failures were analyzed mathematically and statistically to find a minimum corresponding to an optimal maintenance task interval. The proposed models are easy to use and can be implemented during the first three phases of the aircraft's life cycle.*

### Introduction

The operations phase of an aircraft life cycle is the longest, and despite the revenue aircrafts generate for an economy, the average operational cost may exceed the initial purchase price by as much as 10-20 times; maintenance, repair, and overhaul are estimated to be about 10-20% of operational costs [1]. Furthermore, according to the International Air Transport Association (IATA), global maintenance, repair, and overhaul expenditure is expected to increase at an annual growth rate of 4.1%; therefore, airlines are constantly searching for ways to decrease these expenses without compromising on airworthiness [2]. This justifies the need for realism in mathematical models and the way optimization problem is formulated from the design phase of the aircraft lifecycle; system reliability, maintenance processes, and cost must be considered.

Most literature on optimal maintenance task models use the maintenance cost rate as an optimization criterion but overlook the reliability performance. Reducing the system maintenance cost rate does not imply that the system reliability performance is optimized in terms of cost, specifically for multicomponent systems. Minimal maintenance cost is sometimes associated with reduced system reliability measures; this is one of the outcomes of having different components in the system, which may have various maintenance costs and different importance to the system. Considering that an aircraft consists of various systems and components, an optimal maintenance task interval should always consider both the maintenance cost and reliability [3]; this is the motivation for introducing the cost-adjusted importance measure to determine the optimal maintenance task interval.

## Methodology

A significant percentage of maintenance costs is attributed to failures of aircraft components and systems. These failures are random and provide a database that can further be analyzed to aid decision-making for maintenance optimization. Maintenance optimization tasks of aircraft components, subsystems, systems, or structures can be conducted based on analytical, numerical or simulation methods. The analytical method is based on the determination of exact equation; the numerical approach is based on evolutionary methods, descent methods and pattern search methods; the simulation approach is based on Monte-Carlo methods [4-5]; this study applies all three approaches.

To determine an optimal aircraft maintenance task interval, the average operational cost per unit time $E(C/t_M)$ is considered a measure of efficiency; $C$ refers to the operational costs and $T_M$ is the maintenance task interval.

$$E(C/T_M) = \varphi\ (C_R, C_M, T_M, \mathrm{n}, \mathrm{f(t)}) \tag{1}$$

where $C_R$ is the corrective maintenance cost, $C_M$ is preventive maintenance cost, $f(t)$ is the Probability Density Function (PDF) of Time Between Failures (TBF) and $n$ is the number of observed faults/failures. Based on operational experience and documentation, the priori information on $C_R$, $C_M$ and $f(t)$ is known.

Exponential, Erlang, Weibull, Gaussian, inverse Gaussian, and Birnbaum-Saunders are generally applied to model the failure process and deterioration of aircraft engineering items. If the TBF of aircraft components, subsystems, systems, and structures is determined using exponential the PDF is expressed as follows:

$$f(t) = \lambda e^{-\lambda t},\ \lambda > 0,\ t > 0,$$

where λ is failure rate.

The number of failures quantity is described using Poisson distribution

$$P(n/t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}.$$

The expected number of failures during observed interval $T_M$

$$E(n/T_\mathrm{M}) = \lambda T_\mathrm{M}.$$

Equation (1) can be expressed as follows

$$E(C/T_\mathrm{M}) = \lambda C_\mathrm{R} + \frac{C_\mathrm{M}}{T_\mathrm{M}}. \tag{2}$$

With an increase in $T_M$, efficiency decreases, and an optimum point doesn't exist. Therefore, it is impossible to develop an optimal maintenance strategy because $T_M \to \infty$.

If the TBF of aircraft components, subsystems, systems, and structures is determined using exponential the PDF is expressed as follows:

$$f(t) = \lambda^2 t e^{-\lambda t},\ \lambda > 0,\ t > 0.$$

The PDF for the time moment of $n$-th failure

$$f_n(t) = \int_{-i\infty}^{i\infty} \left( \int_0^\infty \lambda^2 t e^{-\lambda t} e^{iwt} dt \right)^n dw. \tag{3}$$

After mathematical simplification, (3) is expressed as

$$f_n(t) = \frac{t^{2n-1}}{(2n-1)!}\lambda^{2n} e^{-\lambda t}.$$

The probability of occurrence of *n* failures during the observed interval is given as

$$F_n(t) = \int_0^t f_n(t)dt. \tag{4}$$

The distribution of failures is calculated as follows:

$$P(n/t) = F_n(t) - F_{n+1}(t) = \int_0^t f_n(t)dt - \int_0^t f_{n+1}(t)dt = \frac{(\lambda t)^{2n+1}}{(2n+1)!}e^{-\lambda t} + \frac{(\lambda t)^{2n}}{(2n)!}e^{-\lambda t}$$

The expected number of failures during the observed interval $T_M$ is expressed as\

$$E(n/T_{\mathrm{M}}) = \sum_{n=1}^{\infty} nP(n/T_{\mathrm{M}}) = \sum_{n=1}^{\infty}\left( n\frac{(\lambda T_{\mathrm{M}})^{2n+1}}{(2n+1)!} + \frac{(\lambda T_{\mathrm{M}})^{2n}}{(2n)!} \right)e^{-\lambda T_{\mathrm{M}}} = \frac{\lambda T_{\mathrm{M}}}{2} + \frac{e^{-2\lambda T_{\mathrm{M}}}}{4} - \frac{1}{4}$$

Equation (1) can be presented as

$$E(C/T_{\mathrm{M}}) = \frac{(2\lambda T_{\mathrm{M}} + e^{-2\lambda T_{\mathrm{M}}} - 1)C_{\mathrm{R}} + 4C_{\mathrm{M}}}{4T_{\mathrm{M}}}. \tag{5}$$

To analyze the dependence (5) for minimum values, the derivative is calculated

$$\frac{dE(n/T_{\mathrm{M}})}{dt} = \frac{-2\lambda C_{\mathrm{R}} T_{\mathrm{M}} e^{-2\lambda T_{\mathrm{M}}} - C_{\mathrm{R}} e^{-2\lambda T_{\mathrm{M}}} + C_{\mathrm{R}} - 4C_{\mathrm{M}}}{T_{\mathrm{M}}^{\ 2}}.$$

The optimal aircraft maintenance task interval is determined by solving the equation

$$-2\lambda C_{\mathrm{R}} T_{\mathrm{M}} e^{-2\lambda T_{\mathrm{M}}} - C_{\mathrm{R}} e^{-2\lambda T_{\mathrm{M}}} + C_{\mathrm{R}} - 4C_{\mathrm{M}} = 0. \tag{6}$$

In this case the approximate formula can be applied

$$e^{-2\lambda T_{\mathrm{M}}} \approx 1 - 2\lambda T_{\mathrm{M}}$$

Therefore,

$$-2\lambda C_{\mathrm{R}} T_{\mathrm{M}} + 4\lambda^2 C_{\mathrm{R}} T_{\mathrm{M}}^{\ 2} - C_{\mathrm{R}} + 2\lambda C_{\mathrm{R}} T_{\mathrm{M}} + C_{\mathrm{R}} - 4C_{\mathrm{M}} = 0$$

$$\lambda^2 C_{\mathrm{R}} T_{\mathrm{M}}^{\ 2} - C_{\mathrm{M}} = 0.$$

The optimal aircraft maintenance task interval

$$T_{\mathrm{M\,opt}} = \sqrt{\frac{C_{\mathrm{M}}}{\lambda^2 C_{\mathrm{R}}}}. \tag{7}$$

Equation (6) is an approximate, the exact formula for the optimal aircraft maintenance task interval can be obtained solving equation (7) based on Lambert function *W(x)*

$$T_{\mathrm{M\,opt}} = \frac{-1 - W\left(\frac{4C_{\mathrm{M}}}{C_{\mathrm{R}}} - 1\right)}{2\lambda}$$

**Analysis and results**

The methodology for finding the optimal aircraft maintenance task interval is shown in Fig. 1.

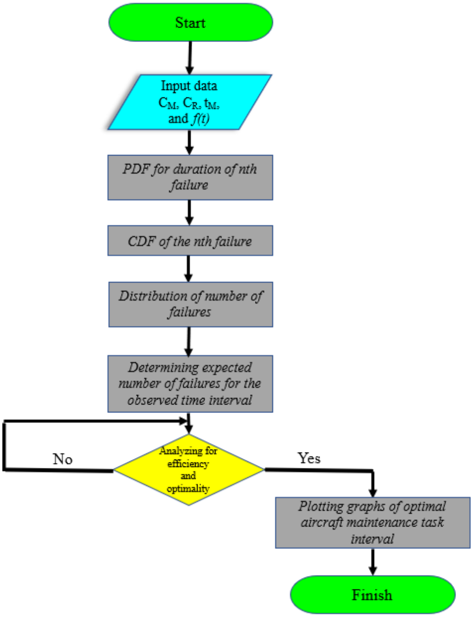
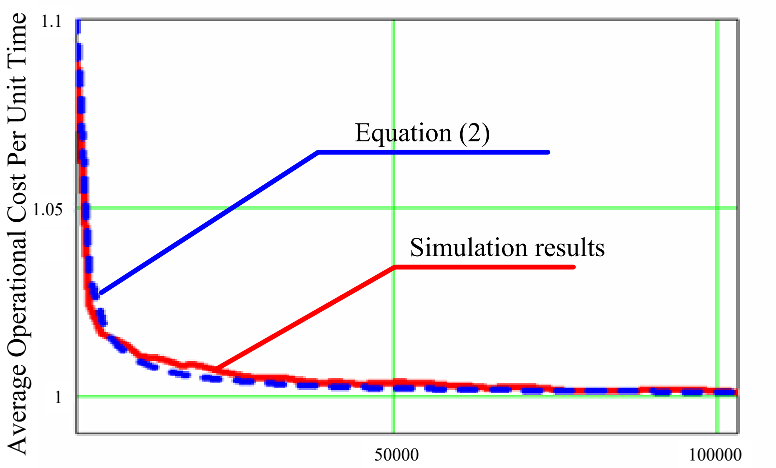



Fig. 1. Algorithm for finding an optimal aircraft maintenance task interval

To analyze the proposed approach, the initial data for simulation are:

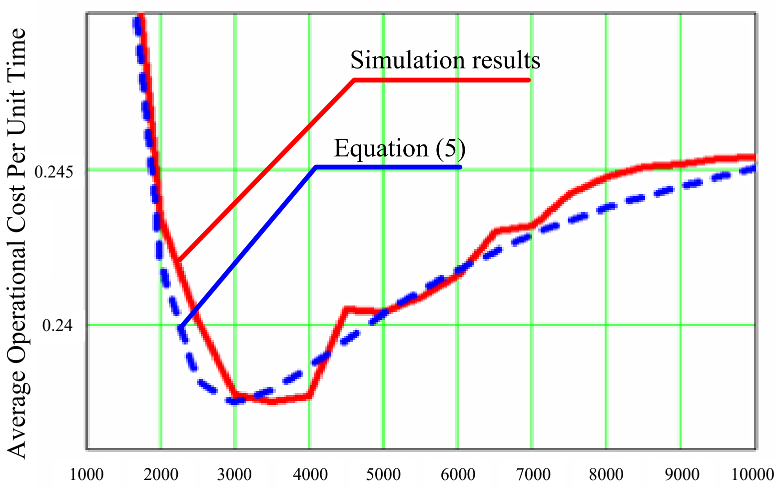
– failure rate $\lambda = 0.0008$ *hours* $^{-1}$
– $C_M = 100$
– $C_R = 800$
– sample size $n=1000$
– number of iterations $N = 10000$

Fig. 2 - 3 show the dependencies of efficiency on optimal aircraft maintenance task interval obtained in accordance with the equation (2) and statistical simulation for initial data set




Maintenance Task Interval

Fig. 2. The dependence of efficiency on aircraft maintenance task interval obtained in accordance with analytical equation (blue line) and statistical simulation (red line) for exponential TBF




Maintenance Task Interval

Fig. 3. The dependence of efficiency on aircraft maintenance task interval obtained in accordance with analytical equation (blue line) and statistical simulation (red line) for Erlang TBF

**Conclusion**

In this study, an estimation of the optimal maintenance task interval of the aircraft component or system is carried out using average operational cost as a measure of efficiency. Two reliability models, the exponential and Erlang models of time between failures were analyzed for optimality using average operational cost per unit time as the efficiency indicator. Analytical equations and statistical simulations of both models show that an optimal aircraft maintenance task interval does not exist for the exponential model because no minimum exists, and the optimal maintenance task interval tends to infinity. However, a minimum exists for the Erlang model, which corresponds to an optimal maintenance task interval. The simulation results coincide with the analytical results; this proves that it is possible to optimize the maintenance task interval of aircraft systems using the Erlang model

The proposed model can be implemented during the first three phases of the aircraft life cycle and can serve as part of a framework for predictive aircraft maintenance.

**References**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